



Digital piracy and firms' strategic interactions: The effects of public copy protection and DRM similarity

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ABSTRACT

The purpose of this paper is to investigate how different types of strategic interaction affect firms' optimal levels of digital rights management (DRM). In our game-theoretical duopoly model, the firms do not directly compete with prices, but they become interdependent while coping with digital piracy. Our analysis shows that (1) stricter public copy protection by the government leads to lower DRM levels and more piracy when the firms regard their DRM levels as "strategic substitutes," but to higher DRM levels and less piracy when the firms perceive their DRM levels as "strategic complements," and (2) a higher degree of similarity between the DRM systems leads to lower DRM levels and more piracy. We also discuss the policy implications of these findings.

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1. Introduction

Digital products, such as CD-ROMs, downloadable software, and e-books, can be easily copied. To protect the digital contents from illegal duplication and distribution, content providers have developed specialized technologies. These protection technologies, such as encryption and copy controls, are collectively termed "digital rights management (DRM)." DRM curbs illegal reproduction but also makes a legal copy less attractive to the users (Wingfield and Smith, 2007). In addition, most of the DRM systems are not effective enough to eradicate illegal use of digital products.

Not surprisingly, copy protection is a controversial issue in the entertainment and software industries. In an online essay, Steve Jobs, CEO of Apple Inc., contends that the major music companies should consider allowing content distributors to sell songs without DRM software since the current DRM systems do not prevent piracy effectively (Jobs, 2007). While his arguments have been embraced gradually by some of the major music companies, DRM is still widely used in the music industry.

The movie industry claims great effects of piracy as well. According to a study conducted by LEK Consulting, in 2005 the worldwide movie industry lost \$18.2 billion to piracy, and US movie studios lost about \$6.1 billion to piracy (McBride and Fowler, 2006). Major US movie studios are trying to cut the loss by embedding copy protection software in their DVD products, and by lobbying the US government to pressure piracy-rampant

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countries for a more aggressive crackdown (King, 2007).

The purpose of this paper is to investigate how different types of strategic interaction between firms affect their optimal protection levels. Our investigation is based on a duopoly setting with two sellers¹ (or content providers) of two digital products. Following Johnson (1985) and others, we develop the model in such a way that the firms do not necessarily compete in terms of prices but are still interdependent in their efforts against piracy. With this model, we identify three types of strategic interaction: strategic substitutes (SS), strategic complements (SC), and a no strategic interaction (NSI) equilibrium. The equilibrium protection levels and prices depend upon two exogenous parameters: public copy protection by the government and dissimilarity between the DRM technologies of the two firms.

When the protection technologies are quite different, successful cracking of one system does not help crack the other system. Given this *low synergy effect* in piracy, one firm's optimal protection level does not depend on the choice of the other firm. We term this case the "no strategic interaction equilibrium." At the intermediate degree of DRM similarity, the protection levels of the two firms tend to move in the same direction. We call this case the "strategic complements equilibrium." At the high degree of DRM similarity, we observe another situation, termed the "strategic substitutes equilibrium," where the protection levels move in the opposite directions.²

Our comparative statics reveals that when the firms consider their DRM levels as strategic substitutes, stricter copy protection by the government may have a crowding-out effect on the private protection level, and thus may be pro-piracy rather than anti-piracy. However, when the firms treat their DRM levels as strategic complements, stricter public protection leads to higher DRM levels and less piracy. Concerning the compatibility of different DRM technologies, we find that increasing similarity between DRM systems tends to lower the private protection levels and thus make piracy more rampant, for both the strategic substitutes and complements equilibria. This result suggests that if policymakers direct digital-goods firms to adopt more similar DRM systems, in order to increase compatibility between contents and playback devices, piracy will become more widely spread as a byproduct.

We also compare our duopoly outcome with the case of a multiproduct monopoly in order to draw a conclusion on social welfare issues. Interestingly, the monopoly may have lower prices and lower DRM levels, relative to the duopoly, and thus improve social welfare.

Our study is based on previous studies in the piracy literature, but features a distinctive perspective on firm-level copy protection efforts. Early research on piracy focuses on photocopying and the issue of how publishers can appropriate some of their lost revenues (e.g., Liebowitz, 1985). Over time the literature has been expanded to include

studies on how copyright protection affects the level of piracy, pricing, development incentives, and social welfare (e.g., Bae and Choi, 2006; Besen and Raskind, 1991; Cremer and Pestieau, 2009). The literature has also examined issues of copyright enforcement, type of appropriation, network effects, and the role of consumer information. Peitz and Waelbroeck (2006) provide a comprehensive review of the piracy literature. They show that most studies can be divided into two groups: a single-product market approach and a multi-product market approach. Our study takes the multi-product approach and is closely related to Johnson (1985) and Belleflamme and Picard (2007) in that the interdependence between the firms comes from their strategies against the piracy rather than from competition in prices.

A major difference between our study and other duopoly-based studies is that the copyright protection efforts of the firms are endogenous variables in our model. Other studies usually focus on how piracy affects prices and profits (e.g., Belleflamme and Picard, 2007; Park and Scotchmer, 2005), while treating the firm-level copyright protection as exogenous. The price of a digital product is apparently an important determinant of the piracy level. With price alone, however, we cannot explain such important questions as why some firms offer their products with a minimum level of protection while their competitors opt for a much higher level.

The remainder of this paper is organized as follows. In Section 2, we develop models with different types of strategic interaction and derive the equilibrium for each type. In the next section, we present and discuss the results of comparative statics of the effects of stricter government protection and higher degree of dissimilarity between the DRM systems of the firms. The paper concludes with a summary and some remarks on future research.

2. Model

2.1. Overview of the model

Our model consists of two firms selling two digital products (products 1 and 2), and the consumers who choose from legal and illegal copies of the products. The number of consumers is normalized to 1, and they are uniformly distributed on the unit interval $[0, 1]$. Consumer i 's identity x_i measures his relative preference over the two products: highest at $x_i = 0$ for product 1, and at $x_i = 1$ for product 2. All consumers have the same base valuation of v for both products. However, they also get *disutility* that is proportional to x_i . For consumer i , the disutility is tx_i when he consumes product 1 instead of his ideal but unavailable product, and $t(1 - x_i)$ when he consumes product 2. The utilities from the two products are, therefore, $v - tx_i$ and $v - t(1 - x_i)$, respectively.

The two products are "piratable": they are imperfectly protected and some consumers are willing to and able to make illegal copies. A user of an illegal copy may be caught by the authority with the probability of α , with $0 < \alpha < 1$. The punishment is confiscation of the illegal copy, and those consumers get zero utility. Therefore, the expected

¹ In this paper, we use content providers, sellers, and firms interchangeably to refer to the producers of digital products (e.g., music and movie studios), online and offline retailers, and rental stores.

² For more details about strategic complements and substitutes, see Bulow et al. (1985).

Table 1
Net utilities from different pairs of products 1 and 2: $u(x_i; A_1, A_2)$.

	$A_1 = B$	$A_1 = C$	$A_1 = O$
$A_2 = B$	$2v - t - p_1 - p_2$ [[$(2v - 1) - (p_1 + p_2)$]]	$(1 - \alpha)(v - t x_i) - e_1 + v - t(1 - x_i) - p_2$ [[$(2v - 1) - (p_2 + e_1 + \alpha v) + \alpha x_i$]]	$v - t(1 - x_i) - p_2$ [[$v - p_2 - 1 + x_i$]]
$A_2 = C$	$v - t x_i - p_1 + (1 - \alpha)(v - t(1 - x_i)) - e_2$ [[$(2v - 1) - (p_1 + e_2 + \alpha(v - 1)) - \alpha x_i$]]	$(1 - \alpha)(2v - t) - \beta(e_1 + e_2)$ [[$(2v - 1) - \beta(e_1 + e_2) - \alpha(2v - 1)$]]	$(1 - \alpha)(v - t(1 - x_i)) - e_2$ [[$(1 - \alpha)(v - 1) - e_2 + (1 - \alpha)x_i$]]
$A_2 = O$	$v - t x_i - p_1$ [[$v - p_1 - x_i$]]	$(1 - \alpha)(v - t x_i) - e_1$ [[$(1 - \alpha)v - e_1 - (1 - \alpha)x_i$]]	0

Note: Net utilities in brackets are with $t = 1$. B, C, and O stand for “buy,” “copy,” and “do not use,” respectively.

utility of an illegal copy of product 1 is $(1 - \alpha)(v - tx_i) + \alpha \cdot 0$ (Yoon, 2002; Bae and Choi, 2006). In this sense, α represents the level of public copy protection by the government (hereinafter referred to as “public protection”) and is exogenous to the firms.

Consumers pay reproduction cost to make an illegal copy (Yoon, 2002). This includes the physical cost (e.g., CD-ROMs to hold copied contents) and the search and learning cost to find hacking technologies (“search cost” hereinafter). Since the physical cost is almost negligible these days, the reproduction cost generally means the search cost. We posit that the search cost is determined by the level of DRM. So, the same notation (e_j) is used for both the search cost and the DRM level.

If a consumer succeeds in the reproduction of a product, then the other product may be reproduced at a lower search cost. This learning effect, or synergy effect, is restricted by the dissimilarity between the DRM systems (β). We assume that the search cost is $\beta(e_1 + e_2)$ when both products are reproduced, and it is smaller than the sum of the two search costs ($e_1 + e_2$), or equivalently, $\beta < 1$. Additionally, it is assumed that the two DRM systems, consisting of one or more protection technologies, are determined before each firm chooses the optimal DRM level (e_j) to be embedded in the product, such as the number of devices a song can be played on. That is, β is predetermined and known to the firms.

Let $u(x_i; A_1, A_2)$ denote the net utility for consumer i after deducting the costs that are paid to buy or copy the products. A_j is the consumer’s choice of product j from the opportunity set of $A = \{B, C, O\}$, where the abbreviations are for buy, copy and do not use, respectively. Let p_j denote the price of product j . Then, we can derive the net utilities in terms of $(v, t, p_1, p_2, \alpha, \beta, e_1, e_2, x_i)$ for all possible pairs of (A_1, A_2) . The complete set of $u(x_i; A_1, A_2)$ is in Table 1.

2.2. Assumptions

We derive the demand functions and the equilibria based on three assumptions. These assumptions are adopted to focus our analysis on more realistic and interesting cases, or to make it tractable.

Assumption 1. $u(x_i; C, C) > u(x_i; B, B)$ for all consumers in $0 \leq x_i \leq 1$.

Using Table 1 with $t = 1$, we obtain

$$u(x_i; C, C) > u(x_i; B, B) \iff p_1 + p_2 > (2v - 1)\alpha + \beta(e_1 + e_2), \tag{1}$$

which means that (B, B) always costs more than (C, C) . In our model both $u(x_i; B, B)$ and $u(x_i; C, C)$ do not depend on, as shown in Table 1. Therefore, if $u(x_i; B, B) > u(x_i; C, C)$ for consumer i , it means that every consumer prefers (B, B) and no one selects (C, C) . The role of Assumption 1 is to rule out this case. This assumption is likely to be satisfied in the markets where the level of public protection (α) or DRM dissimilarity (β) is low, and consequently piracy is widespread. Since Assumption 1 implicitly imposes a restriction on the endogenous variables, i.e., the prices and private protection levels, we check later if it holds in the equilibrium for a reasonable range of the parameters.

Assumption 2. For firm j , the total cost of private copy protection is $K + \frac{m}{2} \{e_j\}^2$.

K is the fixed cost for the DRM system. The marginal cost of raising the private protection level (DRM level) is me_j , with $m > 0$ for increasing marginal cost of e_j .

Assumption 3

$$0 < \alpha < 1, \quad \frac{1}{2} \leq \frac{\max\{e_1, e_2\}}{e_1 + e_2} < \beta < 1, \quad 0 < v, \quad t = 1, \quad 0 < m.$$

Assumption 3 summarizes the restrictions that we make on the parameters. α is the probability of being punished for piracy, and we rule out the unlikely case of being punished always ($\alpha = 1$) or never punished ($\alpha = 0$). β is the index for DRM dissimilarity. It determines how much the search cost is saved when both products are copied: $(e_1 + e_2) - \beta(e_1 + e_2)$. We assume that this synergy effect exists: $(e_1 + e_2) - \beta(e_1 + e_2) > 0 \implies \beta < 1$. To be realistic, however, we also assume that copying one product costs less than copying both even with the synergy effect: $\max\{e_1, e_2\} < \beta(e_1 + e_2) \implies \max\{e_1, e_2\} / (e_1 + e_2) < \beta$. In a symmetric equilibrium where $e_1 = e_2$, the restriction becomes $1/2 < \beta < 1$. The base valuation v is the utility from an ideal good and assumed to be positive. When good 1 is not ideal, the utility ignoring the cost of purchase or reproduction is $v - tx_i$. Since v may take any positive number, we normalize t to 1 without loss of generality. Finally, the marginal cost of raising the protection level is me_j . If $m \leq 0$, the marginal cost is zero or negative and the firms will raise the protection levels until piracy is eliminated completely. The assumption of $m > 0$ is to exclude this unrealistic case.

Given that (B, B) is ruled out by Assumption 1, the demand for product 1 (q_1) is determined by the number of consumers who prefer the remaining options of buying

product 1: (B, O) or (B, C). Among the consumers on the unit interval, $x_i \in [0, 1]$, those with lower x_i have higher relative preference for product 1, hence the *potential buyers* of product 1. We derive q_1 focusing on the low- x_i consumers who will choose from the following five options:

- Buy product 1: (B, O) or (B, C),
- Copy product 1: (C, O), (C, C), or (C, B).

The first step is to identify the group of consumers who prefer (B, O) to (B, C), and another group who prefer (C, O) to (C, C). Let $\hat{x}(BOBC)$ be the consumer who is indifferent between (B, O) and (B, C): $u(x_i; B, O) = u(x_i; B, C)$ if $x_i = \hat{x}(BOBC)$. Similarly, $\hat{x}(COCC)$ is the consumer indifferent between (C, O) and (C, C): $u(x_i; C, O) = u(x_i; C, C)$ if $x_i = \hat{x}(COCC)$. With the utility functions in Table 1, we find that

$$u(x_i; B, O) > u(x_i; B, C) \iff x_i < \hat{x}(BOBC) = \frac{e_2}{1-\alpha} + (1-\nu), \tag{2}$$

$$u(x_i; C, O) > u(x_i; C, C) \iff x_i < \hat{x}(COCC) = \frac{e_2}{1-\alpha} + (1-\nu) - \frac{(1-\beta)(e_1+e_2)}{1-\alpha}. \tag{3}$$

Since $\beta < 1$ and $\alpha < 1$ by Assumption 3, we always have $(1-\beta)(e_1+e_2)/(1-\alpha) > 0$ in Eq. (3). Then, Eqs. (2) and (3) imply that

$$\hat{x}(COCC) = \hat{x}(BOBC) - \frac{(1-\beta)(e_1+e_2)}{1-\alpha} < \hat{x}(BOBC). \tag{4}$$

The two critical values divide the consumers into three groups: GROUPs I–III. All consumers in GROUP I have $x_i \leq \hat{x}(COCC) < \hat{x}(BOBC)$. This group prefers (C, O) to (C, C) as shown in Eq. (3). Also, because $x_i < \hat{x}(BOBC)$ for all consumers in GROUP I, (B, O) is preferred to (B, C) as shown in Eq. (2). Therefore, out of the five options, (C, C) and (B, C) are ruled out and the remaining options for GROUP I are {(C, O), (B, O), (C, B)}. GROUP II lies between the two thresholds: $\hat{x}(COCC) < x_i \leq \hat{x}(BOBC)$. The consumers in this group choose from {(C, C), (B, O), (C, B)} since (C, O) is dominated by (C, C), and (B, C) dominated by (B, O). Consumers in GROUP III have $x_i > \hat{x}(BOBC) > \hat{x}(COCC)$. They choose from {(C, C), (B, C), (C, B)} since (B, O) is dominated by (B, C), and (C, O) by (C, C). The properties of these three groups of potential buyers of product 1 are summarized in Table 2.

2.3. Three types of strategic interaction

Now, we derive the demand function, optimal price and protection level (q_1, p_1, e_1) for each type of strategic interaction, given e_2 .

2.3.1. No strategic interaction (NSI) equilibrium

We have the NSI type of strategic interaction if (p_1, e_1) do not depend on (p_2, e_2) . This type of interaction occurs when firm 1 targets GROUP I for sales. Since the consumers in this group choose from {(C, O), (B, O), (C, B)} as shown in Table 2, the demand for product 1 is the number of consumers who choose (B, O) rather than (C, O) or (C, B). We find that if firm 1 sets (p_1, e_1) such that (B, O) is preferred to (C, O) for some consumers in GROUP I, but for no one in other groups, then q_1 is the same as the number of those consumers who prefer (B, O) to (C, O) because these consumers also prefer (B, O) to (C, B).

To prove this, we first identify the consumer who is indifferent between (B, O) and (C, O) : $x_i = \hat{x}^{NSI} \equiv \hat{x}(BOCO)$. With the utilities in Table 1, we obtain

$$u(x_i; B, O) > u(x_i; C, O) \iff x_i < \hat{x}^{NSI} = \frac{\nu\alpha + e_1 - p_1}{\alpha}. \tag{5}$$

So, all consumers with $x_i < \hat{x}^{NSI}$ prefer (B, O) to (C, O). If \hat{x}^{NSI} falls in GROUP I, i.e.,

$$\hat{x}^{NSI} \leq \hat{x}(COCC) \leq \hat{x}(BOBC), \tag{6}$$

then (B, O) is also preferred to (C, B) for all $x_i < \hat{x}^{NSI}$ because

$$\begin{aligned} u(x_i; C, B) &= u(x_i; C, O) + u(x_i; O, B) && \text{by Table 1,} \\ &< u(x_i; B, O) + u(x_i; O, B) && \text{since } u(x_i; B, O) > u(x_i; C, O) \\ &&& \text{by (5),} \\ &= u(x_i; B, B) && \text{by 1,} \\ &< u(x_i; C, C) && \text{by Assumption 1,} \\ &< u(x_i; C, O) && \text{if } x_i \text{ is in GROUP I (see 2),} \\ &< u(x_i; B, O) && \text{by Eq. (5).} \end{aligned}$$

The demand function for product 1 in the NSI equilibrium is, then,

$$q_1^{NSI}(p_1^{NSI}, e_1^{NSI}) = \hat{x}^{NSI} = \frac{1}{\alpha}(\nu\alpha + e_1^{NSI} - p_1^{NSI}). \tag{7}$$

Note that q_1^{NSI} does not depend on (p_2, e_2) , hence no strategic interaction between the firms if GROUP I is the target group.

Given the demand function (7), firm 1 maximizes

$$\pi_1^{NSI} = p_1^{NSI} \cdot q_1^{NSI}(p_1^{NSI}, e_1^{NSI}) - \left[K + \frac{m}{2} \{e_1^{NSI}\}^2 \right]. \tag{8}$$

By solving this maximization problem, we obtain the optimal price and DRM level as

$$\{p_1^{NSI*}, e_1^{NSI*}\} = \left\{ \frac{\nu\alpha^2 m}{2\alpha m - 1}, \frac{\nu\alpha}{2\alpha m - 1} \right\}. \tag{9}$$

Table 2
Three groups of potential buyers of product 1.

	Consumer ID	Preference	Options
GROUP I	$x_i \leq \hat{x}(COCC) < \hat{x}(BOBC)$	$u(x_i; C, O) \geq u(x_i; C, C)$ $u(x_i; B, O) > u(x_i; B, C)$	(C, O), (B, O), (C, B)
GROUP II	$\hat{x}(COCC) < x_i \leq \hat{x}(BOBC)$	$u(x_i; C, O) < u(x_i; C, C)$ $u(x_i; B, O) \geq u(x_i; B, C)$	(C, C), (B, O), (C, B)
GROUP III	$\hat{x}(COCC) < \hat{x}(BOBC) < x_i$	$u(x_i; C, O) < u(x_i; C, C)$ $u(x_i; B, O) < u(x_i; B, C)$	(C, C), (B, C), (C, B)

{{(C,C), (B,C), (C,B)}, GROUP II from {{(C,C), (B,O), (C,B)}, and GROUP I from {{(C,O), (B,O), (C,B)}. Therefore, the demand for product 1 is determined by the number of consumers who choose (B,C) in GROUP III and (B,O) in GROUP I and II. We find that if firm 1 sets (p_1, e_1) such that (B,C) is preferred to (C,C) for some consumers in GROUP III as well as the entire GROUP I and II, then q_1 is the same as the number of those consumers, who prefer (B,C) to (C,C), because all such consumers choose either (B,C) or (B,O).

To facilitate the proof of the above claim, we first identify the consumer who is indifferent between (B,C) and (C,C), i.e., the consumer with $x_i = \hat{x}^{SS} \equiv \hat{x}(BCCC)$. Using the utility functions in Table 1, we find that

$$u(x_i; B, C) > u(x_i; C, C) \iff x_i < \hat{x}^{SS} \\ = \frac{v\alpha + \beta(e_1 + e_2) - e_2 - p_1}{\alpha} \quad (13)$$

If firm 1 sets (p_1, e_1) such that \hat{x}^{SS} lies in GROUP III, i.e.,

$$\hat{x}^{SS} \geq \hat{x}(BOBC), \quad (14)$$

the demand function is given as

$$q_1^{SS}(p_1^{SS}, e_1^{SS}, e_2^{SS}) = \hat{x}^{SS} = \frac{v\alpha + \beta(e_1^{SS} + e_2^{SS}) - e_2^{SS} - p_1^{SS}}{\alpha}. \quad (15)$$

We prove it by showing that for all $x_i < \hat{x}^{SS}$, the remaining options of copying product 1, {{(C,O), (C,B)}, are both dominated by one of the options of buying product 1, {{(B,O), (B,C)}. First, we show that (C,O) is dominated by (B,O) for all $x_i < \hat{x}^{SS}$. Note that

$$u(x_i; C, O) + u(x_i; O, C) < u(x_i; C, C) \quad \text{by synergy effect (1),} \\ < u(x_i; B, C) \quad \text{by Eq.(13) } \forall x_i < \hat{x}^{SS}, \\ = u(x_i; B, O) \\ + u(x_i; O, C) \quad \text{by 1.}$$

The above result, i.e., $u(x_i; C, O) + u(x_i; O, C) < u(x_i; B, O) + u(x_i; C, O) + u(x_i; O, C) < u(x_i; B, O) + u(x_i; O, C)$, indicates that

$$u(x_i; C, O) < u(x_i; B, O) \quad \forall x_i < \hat{x}^{SS}. \quad (16)$$

Next, we show that (C,B) is dominated by (B,C) for all $x_i < \hat{x}^{SS}$:

$$u(x_i; C, B) = u(x_i; C, O) + u(x_i; O, B) \quad \text{by 1,} \\ < u(x_i; C, O) + u(x_i; O, C) \quad \text{if } u(x_i; O, C) > u(x_i; O, B), \\ < u(x_i; B, O) + u(x_i; O, C) \quad \text{by Eq.(16),} \\ = u(x_i; B, C) \quad \text{by 1.}$$

The second line of the above proof requires $u(x_i; O, C) > u(x_i; O, B)$ for all $x_i < \hat{x}^{SS}$, which can be verified as follows:

$$u(x_i; B, B) < u(x_i; C, C) \quad \text{by Assumption 1,} \\ < u(x_i; B, C) \quad \text{by Eq.(13),} \\ u(x_i; B, O) + u(x_i; O, B) \\ < u(x_i; B, O) + u(x_i; O, C) \quad \text{by 1,} \\ u(x_i; O, B) < u(x_i; O, C). \quad \text{by subtracting } u(x_i; B, O).$$

Between (B,O) and (B,C), as can be verified with Table 2, consumers in GROUP I and II choose (B,O) whereas those consumers in GROUP III who have $x_i < \hat{x}^{SS}$ choose (B,C).

Given the demand function (15), we can derive the corresponding optimal price and DRM level as a function of e_2 :

$$\{p_1^{SS*}, e_1^{SS*}\} = \arg \max p_1^{SS} \cdot q_1^{SS}(p_1^{SS}, e_1^{SS}) - \left[K + \frac{m}{2} \{e_1^{SS}\}^2 \right] \\ = \left\{ \frac{\alpha m (v\alpha - (1-\beta)e_2^{SS})}{2\alpha m - \beta^2}, \frac{\beta(v\alpha - (1-\beta)e_2^{SS})}{2\alpha m - \beta^2} \right\}. \quad (17)$$

At a symmetric SS equilibrium, the equilibrium quantity is $q_1^{SS*} = v\alpha m / [\beta(1-\beta) + (2\alpha m - \beta^2)]$, which is positive if $2\alpha m - \beta^2 > 0$. Assumption 1 holds if $v < B \equiv [\beta(1-\beta) + (2\alpha m - \beta^2)] / [\beta(1-\beta) + (2\alpha m - \beta^2) + \beta]$, where $0 < B < 1$.

2.4. The reaction functions and optimal DRM levels

By combining the three optimal DRM levels in Eqs. (9), (12) and (17), we can derive the following best-response function:

$$e_1^*(e_2) = \begin{cases} e_1^{SS*}(e_2) = \frac{v\alpha\beta - \beta(1-\beta)e_2}{2\alpha m - \beta^2} & \text{if } e_2 \leq \hat{e}_2 \\ e_1^{SC*}(e_2) = \frac{(1-v)\beta + (2v-1)\alpha\beta + \beta^2 e_2}{2m - \beta^2} & \text{if } \hat{e}_2 \leq e_2 \leq \hat{\hat{e}}_2 \\ e_1^{NSI*} = \frac{v\alpha}{2\alpha m - 1} & \text{if } e_2 \geq \hat{\hat{e}}_2 \end{cases} \quad (18)$$

⁴We can derive the reaction function for firm 2 with the same procedure although it is not shown here. The first row of Eq. (18) indicates that the slope of the reaction function is $-\beta(1-\beta)/(2\alpha m - \beta^2)$ in the SS equilibrium. Given $1/2 < \beta < 1$ by Assumption 3, the slope is negative under the following stability and convergence condition

$$2\alpha m - \beta^2 > 0, \quad (19)$$

and the optimal private protection levels (e_1, e_2) become strategic substitutes. In the SC equilibrium, as shown in the middle row of Eq. (18), the slope of the reaction function is $\beta^2/(2m - \beta^2)$. The slope is positive, and thus the optimal private protection levels are strategic complements, under the following condition:

$$2m - \beta^2 > 0. \quad (20)$$

An example of the SS equilibrium is given by EMI Group, who has recently decided to license its music to online sellers without copy protection (Smith and Vara, 2007), even though, its main competitor, Universal Music Group, is greatly concerned about online piracy. Another example is provided by Paramount, a major movie studio, who has refused to make its movies available on Apple's iTunes due to piracy concerns (Grover, 2007). In contrast, Walt Disney Co. has agreed to license its movies to Apple for iTunes. The strategic interaction in the SS equilibrium helps us understand these firms' contrasting decisions.

The SC equilibrium is illustrated by those companies that sell music or movies through the Internet. In 2006, Roxio CinemaNow announced that it would allow consumers to

⁴ $\hat{\hat{e}}_2 \equiv [v\alpha(1-\beta) + (1-\alpha)(v(3\alpha m - 1) - (2\alpha m - 1))] / [(2\alpha m - 1)\beta]$ can be derived by plugging Eq. (9) into (3). To derive \hat{e}_2 , note that putting the three Eqs. (7), (12) and (16) together, we can obtain a twice-kinked demand function for product 1, which is concave in x_i in the neighborhood of $\hat{x}(COCC)$ and convex in the neighborhood of $\hat{x}(BOBC)$. The transition from the SC equilibrium to the SS equilibrium occurs if $\pi_1^{SS*}(e_1^{SS*}(e_2)) > \pi_1^{SC*}(e_1^{SC*}(e_2))$, from which \hat{e}_2 can be derived.

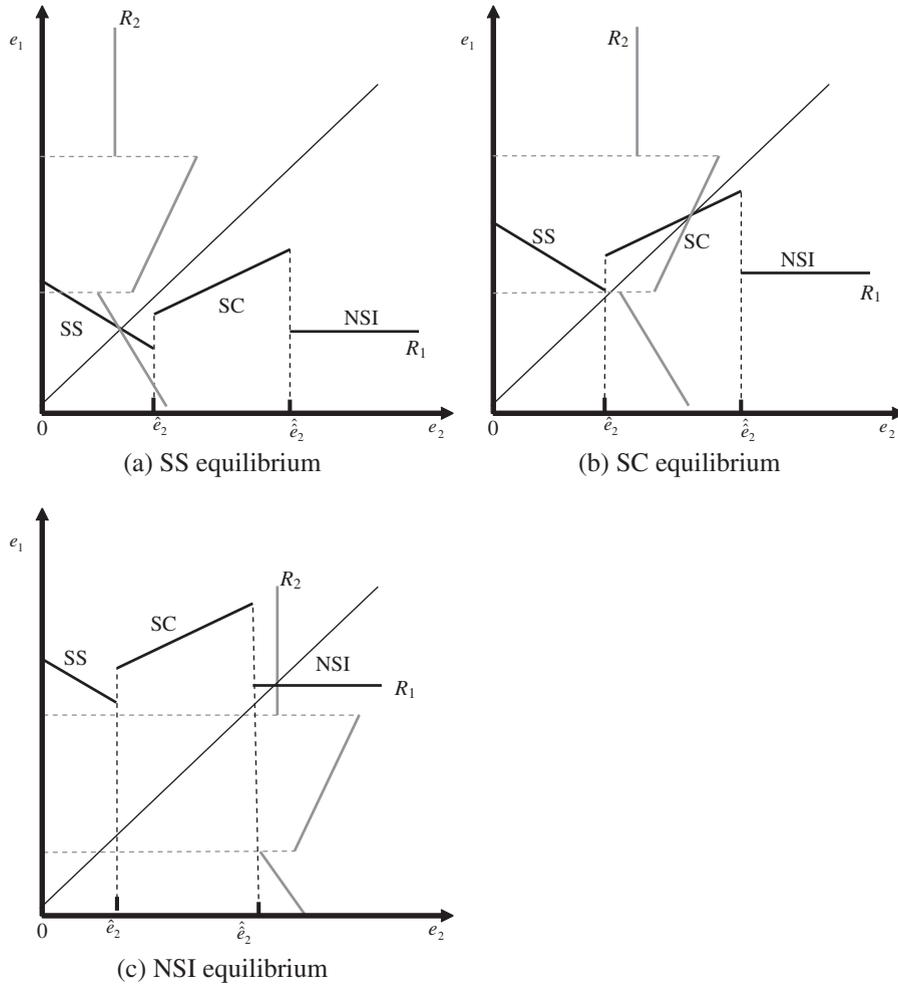


Fig. 1. Reaction functions (R_1, R_2) in three symmetric equilibria.

burn copies of some movies onto a DVD and watch them on TV (McBride, 2006). In response, Movielink, a competing movie-download service, attempted to strike a similar deal with the movie studios. Another example is a recent announcement by Amazon.com that its music-download service would only sell music products that come without copy protection (Smith and Vara, 2007). This announcement was made after Apple’s decision to offer music without copy protection on its iTunes.

Now, we derive the optimal private protection levels. Since the reaction functions are discontinuous, we may not have pure-strategy equilibria. As shown in Fig. 1, the firms reach three different types of equilibria.

Proposition 1. *If condition (19) is satisfied in the SS equilibrium and condition (20) in the SC equilibrium, the optimal DRM levels chosen by firm 1 are*

$$\{e_1^*, p_1^*\} = \begin{cases} \{e_1^{SS^*}, p_1^{SS^*}\} = \left\{ \frac{v\alpha\beta}{2xm+\beta-2\beta^2}, \frac{v\alpha^2 m}{2xm+\beta-2\beta^2} \right\} & \text{if } \beta \leq \hat{\beta} \\ \{e_1^{SC^*}, p_1^{SC^*}\} = \left\{ \frac{(1-v+2\alpha v-\alpha)\beta}{2(m-\beta^2)}, \frac{(1-v+2\alpha v-\alpha)m}{2(m-\beta^2)} \right\} & \text{if } \hat{\beta} \leq \beta \leq \hat{\hat{\beta}} \\ \{e_1^{NSI^*}, p_1^{NSI^*}\} = \left\{ \frac{v\alpha}{2xm-1}, \frac{v\alpha^2 m}{2xm-1} \right\} & \text{if } \beta \geq \hat{\hat{\beta}} \end{cases} \quad (21)$$

2.5. Comparison with a multiproduct monopoly

To compare⁵ the duopoly with a multiproduct monopoly, we consider the case where the two firms merge into a monopoly and set its protection level to maximize the joint profit:

$$\pi_m^R = \pi_1^R + \pi_2^R. \quad (22)$$

In the NSI equilibrium, the optimal level of protection remains the same since there is no strategic interaction. In the SC equilibrium, as seen in Section 2.3.2, the demand for product 1 depends on the number of consumers who prefer (B,O) to (C,C). Similarly the demand for product 2 depends on the number of consumers who prefer (O,B) to (C,C). Thus, (C,C) is a common substitute for (B,O) and (O,B), which makes (B,O) and (O,B) complements. That is, when the monopoly raises the protection level of one product and make (C,C) more costly to consumers, the demand rises for both products, which in turn allows the monopoly to raise the prices. To internalize this

⁵ The interval of β for each type of equilibrium is obtained from Eq. (18). $\hat{\beta}$ is derived from $e_2^{SS^*} \geq \hat{e}_2$ and $\hat{\hat{\beta}}$ is derived from $e_2^{NSI^*} \geq \hat{e}_2$.

positive externality, the monopoly chooses a higher level of protection and price than those of the duopoly in Eq. (21):

$$\{e_m^{SC^*}, p_m^{SC^*}\} = \left\{ \frac{((1-v) + 2\alpha v - \alpha)\beta}{m - 2\beta^2}, \frac{((1-v) + 2\alpha v - \alpha)m}{2(m - 2\beta^2)} \right\}.$$

In the SS equilibrium, the monopoly decreases its protection level and price than those of the duopoly in Eq. (21):

$$\{e_m^{SS^*}, p_m^{SS^*}\} = \left\{ \frac{v\alpha(2\beta - 1)}{2\alpha m - (1 - 2\beta)^2}, \frac{v\alpha^2 m}{2\alpha m - (1 - 2\beta)^2} \right\}.$$

To see why, recall from Section 2.3.3 that the demand in the SS equilibrium depends on the number of consumers who prefer (B,C) to (C,C) for product 1, and consumers who prefer (C,B) to (C,C) for product 2. If the monopoly raises the protection levels, it increases the consumers' costs not just for (C,C) but also for (B,C) and (C,B). With the relatively low β (i.e., low dissimilarity of DRM systems and thus high synergy effect) in the SS equilibrium, some consumers switch to (C,C) from (B,C) or (C,B) to take advantage of the synergy effect of (C,C) and thus save the total cost of consuming the two goods. The reduced demand for both products will result in lower profit. Therefore, the monopoly will decide to lower the protection levels in the SS equilibrium, instead of raising them as in SC equilibrium, and reap the benefits of higher demand and profit. It is notable that the monopoly lowers $e_m = e_1 = e_2$ in the SS equilibrium, whereas a lower e_2 is accompanied by a higher e_1 in the duopoly.

Proposition 2. *The multiproduct monopoly is likely to choose similar or identical copy protection systems across products (i.e., smaller β), to save the cost of DRM, and face a SS equilibrium. Then, the monopoly increases social welfare by lowering the levels of private protection and prices than a duopoly, which in turn boosts the usage of legal products.*

Similar findings are reported by Belleflamme and Picard (2007). They propose that perfectly independent products may become complements in some price range where the monopoly sets its price at a lower level than a duopoly to internalize the positive externality generated by the so-called ‘‘Cournot effect.’’ In our model, in contrast, the complementarity of (B,C) and (C,B) in the SS equilibrium stems from the similarity of the copy protection systems of the two products and the products may remain independent at any level of price.

3. Comparative statics

3.1. The effects of enhanced public copy protection

The public copy protection in our model includes Intellectual Property Rights (IPR) protection. Following previous studies (e.g., Yoon, 2002; Bae and Choi, 2006; Novos and Waldman, 1984), we model the increase in IPR protection as an increase in the cost of piracy. Since the optimal

level of private protection is endogenous, unlike in previous literature, we have both direct and indirect effects of public protection on price and quantity:

$$\frac{dp_1^{R^*}}{d\alpha} = \frac{\partial p_1^{R^*}}{\partial \alpha} + \frac{\partial p_1^{R^*}}{\partial e_1} \frac{de_1^{R^*}}{d\alpha} \quad \text{and} \quad \frac{dq_1^{R^*}}{d\alpha} = \frac{\partial q_1^{R^*}}{\partial \alpha} + \frac{\partial q_1^{R^*}}{\partial e_1} \frac{de_1^{R^*}}{d\alpha}. \quad (23)$$

The signs of $\partial p_1^{R^*}/\partial \alpha$, $\partial p_1^{R^*}/\partial e_1$, $\partial q_1^{R^*}/\partial \alpha$, and $\partial q_1^{R^*}/\partial e_1$ can be identified using Eqs. (7), (9), (11), (12), (15), and (17). Stricter private protection increases the copy cost and thus increases the demand, which enables the firm to raise the price: $\partial q_1^{R^*}/\partial e_1 > 0$ and $\partial p_1^{R^*}/\partial e_1 > 0$ where $R = \{NSI, SC, SS\}$. The effect of stricter public protection varies in different types of equilibrium. In the SC equilibrium, it results in an outward parallel shift in demand, as one can verify with Eq. (11), and content providers raise the price and quantity. In SS and NSI equilibria, stricter public protection lowers the slope of the demand curve ($-1/\alpha$), as indicated by Eqs. (7) and (15). With the less elastic demand, the content providers now target only the high-valuation consumers, i.e., sell less quantity, and charge them a higher price.

To see the effect of an increase in public protection on the optimal level of private protection, let us first define $e_1^{R^*} = R_1(\alpha, e_2^{R^*})$ and $e_2^{R^*} = R_2(\alpha, e_1^{R^*})$ as the optimal levels of private protection, where $R^* = \{SS^*, SC^*\}$. Differentiation of the equilibrium condition $e_1^{R^*} = R_1(\alpha, R_2(\alpha, e_1^{R^*}))$ yields

$$\frac{de_1^{R^*}}{d\alpha} = \frac{1}{(1 - \partial R_1/\partial e_2 \cdot \partial R_2/\partial e_1)} \left[\frac{\partial e_1^{R^*}}{\partial \alpha} + \frac{\partial R_1}{\partial e_2} \frac{dR_2}{d\alpha} \right]. \quad (24)$$

At a stable equilibrium with $|\partial R_1/\partial e_2| < 1$ and $|\partial R_2/\partial e_1| < 1$, the denominator on the right hand side of Eq. (24) is always positive. The sign of $de_1^{R^*}/d\alpha$ thus depends on three factors: (1) $\partial e_1^{R^*}/\partial \alpha$ which is the direct effect of α on $e_1^{R^*}$, (2) $\partial R_1/\partial e_2$ which is the slope of the reaction function, and (3) $dR_2/d\alpha$ which is firm 2's response to an increase in α .⁶ Proposition 3 and Table 4 summarize the results of the comparative statics.

Proposition 3. *If the government strengthens the public copy protection,*

- (a) *the firms will decrease their DRM efforts in the SS equilibrium, but increase in the SC equilibrium;*
- (b) *the firms will raise their prices in the SC equilibrium, but the effect in the SS equilibrium is mixed (raise price if β is sufficiently high but lower price otherwise);*
- (c) *fewer consumers will choose to buy (i.e., more will choose to copy the products) in the SS equilibrium, but more consumers will buy in the SC equilibrium.*

Proof. See Appendix A.

The intuition underlying Proposition 3 is as follows. In the SS equilibrium, the direct effect is negative for both firms: $\partial e_1/\partial \alpha < 0$, $\partial e_2/\partial \alpha < 0$. It is, however, partly offset

⁶ $dR_2/d\alpha$ is obtained indirectly by differentiating the reaction function of firm 1 in Eq. (18) with respect to α , which yields $\beta[2(1 - \beta)me_1 - v\beta^2]/[(2\alpha m - \beta^2)^2]$ for the SS equilibrium and $\beta(2v - 1)/(2m - \beta^2)$ for the SC equilibrium.

Table 4
Summary of comparative statics.

	e^R	p^R	q^R
Strategic substitute	$\frac{de^{SS}}{d\alpha} \leq 0, \frac{de^{SS}}{d\beta} \geq 0$	$\frac{dp^{SS}}{d\alpha} \geq 0, \frac{dp^{SS}}{d\beta} \geq 0$	$\frac{dq^{SS}}{d\alpha} \leq 0, \frac{dq^{SS}}{d\beta} \geq 0$
Strategic complement	$\frac{de^{SC}}{d\alpha} \geq 0, \frac{de^{SC}}{d\beta} \geq 0$	$\frac{dp^{SC}}{d\alpha} \geq 0, \frac{dp^{SC}}{d\beta} \geq 0$	$\frac{dq^{SC}}{d\alpha} \geq 0, \frac{dq^{SC}}{d\beta} \geq 0$
No strategic interaction	$\frac{de^{NSI}}{d\alpha} \leq 0$	$\frac{dp^{NSI}}{d\alpha} \geq 0$	$\frac{dq^{NSI}}{d\alpha} \leq 0$

by the strategic decisions of the firms: firm 1 responds to lower e_2 with higher e_1 in the SS equilibrium. As a result, an increase in public protection exerts two opposite effects on the optimal level of DRM, but the net effect is a lower level of DRM.

In the case of the SC equilibrium, an outward-parallel shift of demand following an increase in the public protection level makes the content providers respond with a higher DRM level, which expands the demand further. This effect is further augmented by the strategic complementarity of the firms' decisions in the SC equilibrium. Therefore, a change in the level of the public protection is accompanied by direct and strategic effects on the optimal DRM level in the same direction.

The total effect of a higher level of α on firm 1's profit can be identified by taking the total derivative of π_1^R with respect to α :

$$\frac{d\pi_1^R}{d\alpha} = \frac{\partial \pi_1^R}{\partial \alpha} + \frac{\partial \pi_1^R}{\partial e_2} \frac{de_2^R}{d\alpha}. \quad (25)$$

The first term on the right-hand side is the direct effect, and the second term is the strategic effect caused by the response of firm 2 to the change in α . The firm always prefers a higher level of public protection since direct and strategic effects on the profit are both positive in both types of equilibrium. However, the components of the strategic interactions are different: $\partial \pi_1 / \partial e_2$ and $de_2 / d\alpha$ are both negative in the SS equilibrium, but positive in the SC equilibrium.

Most national governments are concerned about piracy and are taking steps to curb unauthorized reproduction (McCary, 2007). If private protection is insufficient, the government may want to increase the public protection level. However, as shown in Proposition 3, higher level of public protection does not necessarily result in stronger overall protection. In the SS and NSI equilibrium, a higher level of public protection leads companies to reduce their DRM levels, while in the SC equilibrium the DRM levels increase. It follows that careful investigation of the strategic interaction in the market is of critical importance for the policymakers to obtain a desired level of overall protection.

Proposition 3 also reveals that, in the SS and NSI equilibrium, enhanced public protection can cause more consumers to turn to pirated products. In the SC equilibrium, however, an increase in public protection leads to higher demand for legitimate digital goods and thus less piracy. When the objective is to discourage piracy, a policy towards stronger public protection can achieve its goal only in the SC equilibrium. In the SS and NSI equilibrium, the

government should do the opposite, i.e., lower the level of public protection.

3.2. The effects of higher similarity between DRM systems

Another interesting subject of comparative statics is how a higher degree of similarity between DRM systems affects the optimal levels of DRM, the prices, and the quantities. Using the same procedure, we obtain the results presented in Proposition 4 and Table 4.

Proposition 4. *If DRM systems become more similar across firms, in both SS and SC equilibrium,*

- the firms will decrease their DRM efforts;*
- the firms will lower their prices;*
- fewer consumers will buy legitimate goods.*

Proof. See Appendix B.

In 2006, France passed a controversial law that would require sellers of digital-music players, e.g. Apple's iPod, and online music services, e.g. Apple's iTunes, to make their technical standards available to other companies. The purpose was to make different music tracks and playback devices "interoperable" (Hesseldahl, 2006). Other European countries might follow suit (Carlin, 2007). A strong objection was raised on grounds that such laws would lead to "state-sponsored piracy" (Hesseldahl, 2006). In a similar vein, the International Federation of the Phonographic Industry blamed a "culture of state-tolerated apathy toward illegal file-sharing" for the continuous decline in global music sales (Pfanner, 2010).

Proposition 4 shows that more similarity (i.e., lower β) between DRM systems would lower the DRM levels, the prices, and the demand for legal products. Therefore, when the firms are forced to adopt similar DRM systems by law, the demand for legitimate products would decrease even though the prices would be lower (i.e., a positive relationship between price and demand). The higher synergy effect in piracy resulting from the higher DRM similarity would motivate consumers to obtain (free) pirated products, instead of buying legal products, despite the cheaper prices. Our results clearly support the "state-sponsored piracy" argument.

4. Conclusion

This paper examines how different types of strategic interaction between two digital-goods firms influence the

optimal DRM levels and prices. We first develop a model that includes three types of strategic interaction between the firms: strategic substitutes, strategic complements, and no strategic interaction equilibrium. Then, we derive the optimal DRM levels, prices, and demand functions for the legitimate products for each type of strategic interaction. Our analysis offers the following insights. First, stricter public copy protection would lead to reduced firm-level protection and more piracy among consumers in the SS and NSI equilibrium, but higher DRM levels and less piracy in the SC equilibrium. Second, lower degree of dissimilarity between the protection systems lowers DRM levels and increases piracy. Third, when the protection systems are very similar, a monopoly would choose lower DRM levels and lower prices than a duopoly.

We provide some new and important insights for the literature on piracy. Most of the related studies examine monopoly situations, and only a small number of studies deal with duopoly cases (e.g., Johnson, 1985; Park and Scotchmer, 2005). Those studies that do have duopoly settings usually focus on prices of information goods, rather than directly address the issue of how the level of private copy protection, such as DRM, is determined. This paper incorporates the DRM levels chosen by the firms, as well as the prices of digital goods, as the key endogenous variables in a duopoly setting. With this more flexible approach, we have illustrated that, even if firms are independent of each other in the price-setting game, they may become interdependent through DRM competition when a significant portion of consumers prefer pirated goods.

Finally, we offer some suggestions for future research. One possible direction would be to allow the two firms to cooperate or collude with each other, which is in line with the work of Park and Scotchmer (2005). In the current paper, the two content providers set their optimal DRM levels separately. How would the results change if the content providers jointly develop a protection system and share the cost of DRM? Would this be a better approach to dealing with piracy? Would consumers be better off? Would the overall industry be better off? These are important and interesting research questions that need to be addressed. Another direction would be to include content producers and content sellers as separate players in a model. In the current paper, firms produce and sell digital goods to consumers. In reality, however, content producers are not usually content sellers. Music companies and movie studios produce content, but they usually are not engaged in selling their products directly to consumers. Rather, they make their products available to consumers through online and offline retailers. How would this setup change the optimal levels of DRM and prices for digital goods? Research in this direction will be a meaningful extension of this paper.

Appendix A. Proof of Proposition 3

Total differentiation of the optimal levels of private copy protection with respect to α under different equilibrium types yields

$$\frac{de_1^{SS^*}}{d\alpha} = \frac{1}{(1 - \partial R_1/\partial e_2 \cdot \partial R_2/\partial e_1)} \left[\underbrace{\frac{\partial e_1^{SS^*}}{\partial \alpha}}_{(-)} + \underbrace{\frac{\partial R_1}{\partial e_2}}_{(-)} \underbrace{\frac{dR_2}{d\alpha}}_{(-)} \right]$$

$$= -\frac{v(2\beta - 1)\beta^2}{((2\alpha m - \beta^2) + \beta(1 - \beta))^2} \leq 0,$$

$$\frac{de_1^{SC^*}}{d\alpha} = \frac{1}{(1 - \partial R_1/\partial e_2 \cdot \partial R_2/\partial e_1)} \left[\underbrace{\frac{\partial e_1^{SC^*}}{\partial \alpha}}_{(+)} + \underbrace{\frac{\partial R_1}{\partial e_2}}_{(+)} \underbrace{\frac{dR_2}{d\alpha}}_{(+)} \right]$$

$$= \frac{\beta(2v - 1)}{2(m - \beta^2)} \geq 0, \text{ and}$$

$$\frac{de_1^{NSI^*}}{d\alpha} = -\frac{v}{(2\alpha m - 1)^2} \leq 0.$$

With these results, we can determine the effect of α on the equilibrium price and quantity under different equilibrium types.

$$\frac{\partial p_1^{SS^*}}{\partial \alpha} = \underbrace{\frac{\partial p_1^{SS^*}}{\partial \alpha}}_{(+)} + \underbrace{\frac{\partial p_1^{SS^*}}{\partial e_1}}_{(+)} \underbrace{\frac{de_1^{SS^*}}{d\alpha}}_{(-)} = \frac{v\alpha m((2\alpha m - \beta^2) + \beta(2 - 3\beta))}{((2\alpha m - \beta^2) + \beta(1 - \beta))^2} \stackrel{v \geq 0}{\geq} 0,$$

$$\frac{\partial p_1^{SC^*}}{\partial \alpha} = \underbrace{\frac{\partial p_1^{SC^*}}{\partial \alpha}}_{(+)} + \underbrace{\frac{\partial p_1^{SC^*}}{\partial e_1}}_{(+)} \underbrace{\frac{de_1^{SC^*}}{d\alpha}}_{(+)} = \frac{(2v - 1)m}{2(m - \beta^2)} \geq 0,$$

$$\frac{\partial p_1^{NSI^*}}{\partial \alpha} = \underbrace{\frac{\partial p_1^{NSI^*}}{\partial \alpha}}_{(+)} + \underbrace{\frac{\partial p_1^{NSI^*}}{\partial e_1}}_{(+)} \underbrace{\frac{de_1^{NSI^*}}{d\alpha}}_{(-)} = \frac{2v\alpha m(\alpha m - 1)}{(2\alpha m - 1)^2} \stackrel{v \geq 0}{\geq} 0,$$

$$\frac{\partial q_1^{NSI^*}}{\partial \alpha} = \underbrace{\frac{\partial q_1^{NSI^*}}{\partial \alpha}}_{(-)} + \underbrace{\frac{\partial q_1^{NSI^*}}{\partial e_1}}_{(+)} \underbrace{\frac{de_1^{NSI^*}}{d\alpha}}_{(-)} = -\frac{vm}{(2\alpha m - 1)^2} \leq 0,$$

$$\frac{\partial q_1^{SS^*}}{\partial \alpha} = \underbrace{\frac{\partial q_1^{SS^*}}{\partial \alpha}}_{(-)} + \underbrace{\frac{\partial q_1^{SS^*}}{\partial e_1}}_{(+)} \underbrace{\frac{de_1^{SS^*}}{d\alpha}}_{(-)} = \frac{v(1 - 2\beta)\beta m}{(2\alpha m + \beta - 2\beta^2)^2} \leq 0, \text{ and}$$

$$\frac{\partial q_1^{SC^*}}{\partial \alpha} = \underbrace{\frac{\partial q_1^{SC^*}}{\partial \alpha}}_{(+)} + \underbrace{\frac{\partial q_1^{SC^*}}{\partial e_1}}_{(+)} \underbrace{\frac{de_1^{SC^*}}{d\alpha}}_{(+)} = \frac{(2v - 1)m}{2(m - \beta^2)} \geq 0. \quad \square$$

Appendix B. Proof of Proposition 4

Differentiation of the optimal level of private copy protection with respect to β under different equilibrium types yields

$$\frac{de_1^{SS^*}}{d\beta} = \frac{1}{(1 - \partial R_1/\partial e_2 \cdot \partial R_2/\partial e_1)} \left[\underbrace{\frac{\partial e_1^{SS^*}}{\partial \beta}}_{(+)} + \underbrace{\frac{\partial R_1}{\partial e_2}}_{(-)} \underbrace{\frac{dR_2}{d\beta}}_{(+)} \right]$$

$$= \frac{2v\alpha(\alpha m + \beta^2)}{(2\alpha m + \beta - 2\beta^2)^2} \geq 0,$$

$$\frac{de_1^{SC}}{d\beta} = \frac{1}{(1 - \partial R_1/\partial e_2 \cdot \partial R_2/\partial e_1)} \left[\underbrace{\frac{\partial e_1^{SC}}{\partial \beta}}_{(+)} + \underbrace{\frac{\partial R_1}{\partial e_2}}_{(+)} \underbrace{\frac{dR_2}{d\beta}}_{(+)} \right]$$

$$= \frac{((1 - \nu) + (2\nu - 1)\alpha)(m + \beta^2)}{2(m - \beta^2)^2} \geq 0.$$

With the above results, we can determine the effect of β on the equilibrium price and quantity under different equilibrium types.

$$\frac{\partial p_1^{SS*}}{\partial \beta} = \underbrace{\frac{\partial p_1^{SS*}}{\partial \beta}}_{(+)} + \underbrace{\frac{\partial p_1^{SS*}}{\partial e_1}}_{(+)} \underbrace{\frac{de_1^{SS*}}{d\beta}}_{(+)} = \frac{\nu\alpha^2 m(4\beta - 1)}{(2\alpha m + \beta - 2\beta^2)^2} \geq 0,$$

$$\frac{\partial p_1^{SC*}}{\partial \beta} = \underbrace{\frac{\partial p_1^{SC*}}{\partial \beta}}_{(+)} + \underbrace{\frac{\partial p_1^{SC*}}{\partial e_1}}_{(+)} \underbrace{\frac{de_1^{SC*}}{d\beta}}_{(+)} = \frac{((1 - \nu) + (2\nu - 1)\alpha)m\beta}{(m - \beta^2)^2} \geq 0,$$

$$\frac{\partial q_1^{SS*}}{\partial \beta} = \underbrace{\frac{\partial q_1^{SS*}}{\partial \beta}}_{(+)} + \underbrace{\frac{\partial q_1^{SS*}}{\partial e_1}}_{(+)} \underbrace{\frac{de_1^{SS*}}{d\beta}}_{(+)} = \frac{\nu\alpha m(4\beta - 1)}{(2\alpha m + \beta - 2\beta^2)^2} \geq 0, \text{ and}$$

$$\frac{\partial q_1^{SC*}}{\partial \beta} = \underbrace{\frac{\partial q_1^{SC*}}{\partial \beta}}_{(+)} + \underbrace{\frac{\partial q_1^{SC*}}{\partial e_1}}_{(+)} \underbrace{\frac{de_1^{SC*}}{d\beta}}_{(+)} = \frac{((1 - \nu) + (2\nu - 1)\alpha)m\beta}{(m - \beta^2)^2} \geq 0.$$

□

References

Bae, S.H., Choi, J.P., 2006. A model of piracy. *Information Economics and Policy* 18, 303–320.
 Belleflamme, P., Picard, P.M., 2007. Piracy and competition. *Journal of Economics & Management Strategy* 16 (2), 351–383.

Besen, S.M., Raskind, L.J., 1991. An introduction to the law and economics of intellectual property. *Journal of Economic Perspectives* 5, 3–27.
 Bulow, J.I., Geanakoplos, J.D., Klemperer, P.D., 1985. Multimarket oligopoly: strategic substitutes and complements. *Journal of Political Economy* 93 (3), 488–511.
 Carlin, D., 2007. Europe vs. Apple: Facing the Music. *BusinessWeek* (January 31).
 Cremer, H., Pestieau, P., 2009. Piracy prevention and the pricing of information goods. *Information Economics and Policy* 21 (1), 34–42.
 Grover, R., 2007. Why hollywood snubbed jobs at Macworld. *BusinessWeek* (January 12).
 Hesseldahl, A., 2006. Apple vs. France. *BusinessWeek* (March 21).
 Jobs, S., 2007. Thoughts on Music. <www.apple.com/hotnews/thoughtsonmusic> (accessed 15.09.10).
 Johnson, W.R., 1985. The economics of copying. *Journal of Political Economy* 93 (1), 158–174.
 King Jr., N., 2007. U.S. piracy case may raise trade tensions with China. *The Wall Street Journal* (April 7).
 Liebowitz, S.J., 1985. Copying and indirect appropriability: photocopying of journals. *Journal of Political Economy* 93, 945–957.
 McBride, S., 2006. New service lets users burn studio DVDs. *The Wall Street Journal* (July 19).
 McBride, S., Fowler, G.A. 2006. Studios see big rise in estimates of losses to movie piracy. *The Wall Street Journal* (May 3).
 McCary, J., 2007. Courting allies to fight counterfeiters. *The Wall Street Journal* (April 13).
 Novos, I.E., Waldman, M., 1984. The effects of increased copyright protection: an analytic approach. *Journal of Political Economy* 92 (2), 236–246.
 Park, Y., Scotchmer, S., 2005. Digital Rights Management and the Pricing of Digital Products. NBER Working Paper.
 Peitz, M., Waelbroeck, P., 2006. Piracy of digital products: a critical review of the theoretical literature. *Information Economics and Policy* 18 (4), 449–476.
 Pfanner, E., 2010. Music industry counts the cost of piracy. *The New York Times* (January 22).
 Smith, E., Vara, V., 2007. Music service from amazon takes on itunes. *The Wall Street Journal* (May 17).
 Wingfield, N., Smith, E., 2007. Jobs's new tune raises pressures on music firms. *The Wall Street Journal* (February 7).
 Yoon, K., 2002. The optimal level of copyright protection. *Information Economics and Policy* 14 (3), 327–348.