A Model of Piracy

By

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Abstract

This paper develops a simple model of software piracy to analyze the short-run effects of piracy on software usage and the long-run effects on development incentives. We consider two types of costs associated with piracy: the *reproduction cost* that is *constant* across users and the *degradation cost* that is *proportional* to consumers' valuation of the original product. We show that the effects of piracy depend crucially on the nature of piracy costs. Policy implications concerning copyright protection are also discussed.

JEL Classification: O34, L1, L86.

Keywords: copyright protection, piracy, intellectual property, self-selection.

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1. Introduction

As the current controversy surrounding the Napster case testifies, unauthorized reproduction of intellectual property has been a serious but controversial issue for copyright holders, consumers, and policy makers alike, especially with the advent of digital technology. According to a recent study by the Business Software Alliance (2005), for instance, the piracy rate in 2004 is estimated to be 35% which can be translated into $33 billion dollar losses for software publishers.[1] The piracy rates in the Far East, especially China and Vietnam, represent in each case over 90%. The corresponding rates for European Union and North America are 35% and 22%, respectively. Based on these figures, copyright holders claim that piracy is a severe threat to incentives to develop new products, as well as revenue loss from the developed software.

This paper develops a self-selection model of software piracy to analyze the short-run effects of piracy on software usage and the long-run effects on development incentives. The innovation in this paper is to distinguish two types of costs associated with piracy – constant and type-proportional – and to show that the effects of piracy depend crucially on the nature of piracy costs. To comment briefly on the two types of costs, we first assume that piracy entails *reproduction cost*, which is assumed to be *constant* across all consumers. The constant reproduction cost can be treated as the price of an illegal copy made by piracy retailers. Even with the assumption that actual reproduction cost is zero, consumers will need to exert efforts to obtain an authorized copy or may have to spend time to find the software, and then to copy or to download it for installation purposes. We assume that these costs are the same across all users or at least *independently* distributed with the valuation of consumers for the product.

[1] 'Second Annual BSA and IDC global software piracy study (May 2005)' conducted by International Data Corporation (IDP) for the Business Software Alliance (BSA) and its member companies.

The second type of cost we consider is *degradation of product quality* associated with unauthorized copying. The utility loss due to degradation is *proportional* to each consumer's valuation of the software. As examples of degradation of the quality associated with piracy, for instance, an authorized copy of the software is bundled with manuals, installation software, online services, as well as discount on future upgrades. Users who pirate the software, in contrast, may not be able to access the entire complementary bundle. In addition, some software publishers strategically include web-based functions that require personal identification numbers (PIN). Identical PINs cannot be used on the Web at the same time.[2] With an illegal copy, a consumer thus may be able to use the software off-line, but not able to use online functions, thus experiencing quality degradation.[3] The web-based anti-piracy system is also capable of reducing quality of the software by blocking the users with a pirated copy from receiving upgrades.[4]

As another example of policy relevance, consider the Peer to Peer Piracy Prevention Act (H.R. 5211) which was proposed by Rep. Howard Berman, D-California, and is being considered by the House Judiciary subcommittee as a way to protect intellectual property against file-sharing through peer-to-peer (P2P) networks. P2P networks arose as a response to the shutdown of Napster. Unlike Napster, P2P networks do not host files on a central server; instead they list available files on individual PCs and directly connect those computers, which makes the enforcement of copyrights more difficult. The proposed bill would allow the record industry to "hack" into individuals' PCs in search of copyright violations. In our model, this type of copyright enforcement would translate into an increase in the costs of copying that are proportional to user types.

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[2] With development of high-speed Internet, the new components of game software are online game services, in which you can play the game with someone through the Internet. With the database for the registration keys the software publisher is able to prohibit the pirated copies with the duplicated key from running on the web.

[3] For TurboTax software, each authorized copy is designed to file tax returns electronically for only one customer. Hence, users with an unauthorized copy are only able to do tax returns except electronic filing.

[4] Responding to piracy of Windows XP operating system, Microsoft announces that Windows XP Service Pack 1 (and possibly all future updates) will not install with pirated copies.

Another response by record labels in face of file-sharing is to upload the so-called "spoof" files – containing little or no music – on P2P networks to confuse downloaders. This practice would increase the expected time of downloading a particular music file and can be considered as an increase in the uniform copy costs.

The purpose of this study is to construct a model of self-selection with heterogeneous consumers who choose among three available options: the purchase of authorized product, the use of an illegal copy with unauthorized reproduction, or no consumption. In such a framework, we conduct a two-step analysis. In the short-run analysis, we investigate how the threat of piracy constrains the pricing behavior of the monopolist. Depending on the relative magnitudes of reproduction and degradation cost, the monopolist is shown to choose the optimal choice of regimes between limit pricing and accommodation to copy. We demonstrate that with the threat of piracy the monopolist's price is lowered, and usage of an authorized copy is *increased* in both regimes with positive welfare implications.[5] This result provides a sharp contrast to the common claims of copyright holders, in which the possibility of piracy reduces demand for a legal copy. We then conduct a comparative statics exercise that analyzes the effects of increased copyright protection. It is shown that the effects could hinge on how it affects the two margins of the piracy costs discussed above. The reason is that the changes in the two margins impact the demand for a legal copy in different ways: the changes in constant production cost shifts the demand curve in a *parallel* fashion whereas the changes in degradation rate induces a *pivot* change in the demand curve.

In the long-run analysis, we extend the model to allow for an endogenous choice of the software quality by producer. The existence of piracy creates inefficiently low quality of the product. Thus, there is potential for increased copyright protection to help mitigate this inefficiency in quality provision and counterbalance the short-run effects.

[5] Choi and Thum (2002) provide a similar framework, if we consider purchasing an authorized copy of the software as entering the official economy, and making an illegal copy as operating in the shadow economy.

Once again, we demonstrate that the effects of increased copyright protection on the provision of quality depend on how it affects the two margins of the piracy costs.

Earlier papers concerned with the effects of increased copyright protection on social welfare include Novos and Waldman (1984), Johnson (1985), Harbaugh and Khemka (2001), and Yoon (2002) among others.[6] Novos and Waldman (1984) analyze the effects of increased copyright protection in a model where consumers vary only in terms of their cost of obtaining a copy. They show that there is no tendency for an increase in copyright protection to increase the social welfare loss due to underutilization once the cost of obtaining a copy is taken into account. Our paper, in contrast, allows consumers to vary in terms of their valuations on the quality of software. In addition, we consider two different types of costs associated with piracy and shows that the effects of increased copyright protection hinge on how it affects the two margins of the piracy costs.

Johnson (1985), as in our paper, considers consumers with different tastes, but his model is of horizontal differentiation and the focus is on the product variety issues. More importantly, the major difference between his paper and ours is in the long-run analysis: in Johnson's analysis software supply responses are modeled along the extensive margin (the number of software products created), whereas in our analysis supplier responses are modeled along the intensive margin (the quality of software) as in Novos and Waldman (1984). In a recent paper, Belleflamme (2002) analyzes pricing decisions of producers of information goods in the presence of copying. He assumes a uniform distribution of consumer types in a model of vertical differentiation and derives similar results as in our paper. Once again, however, his long-run analysis is along the extensive margin as in Johnson (1985).

Yoon (2002) adopts a similar framework to ours in the analysis of the *usage* phase. The major difference with Yoon (2002), however, lies in the treatment of the

[6] See Peitz and Waelbroeck (2003) for a recent survey on piracy of digital products. For empirical studies of the effect of file sharing on record sales, see Oberholzer and Strumpf (2004) and Hui and Png (2003).

development phase. He derives the optimal level of copyright protection by assuming that the development cost is fixed in his long-run analysis.[7] The only measure for the ex ante efficiency with fixed development cost is whether the monopolist develops the new product or not; the monopolist does not introduce a new product if the development cost can not be recovered due to weak protection level. With this type of ex ante efficiency measure, the optimal protection level is characterized by a step function because the incentive for development can be altered with an infinitesimal change of intellectual property rights protection (IPRP). In our model, to have a continuous effect of the increase in IPRP, we assume that the monopolist's long-run incentive is to choose the quality of software.[8]

Harbaugh and Khemka (2001) make a distinction between broad-based enforcement and targeted enforcement aimed at specific users. They show that the traditional tradeoff between copyright holder profits and consumer surplus hinges on the assumption of broad-based enforcement. If enforcement is targeted at high-valuation buyers such as big corporations, the copyright holder charges super-monopoly prices and induces piracy among low-valuation buyers. In such a case, they show that extending the range of targeted enforcement down the demand curve lowers prices toward the monopoly level, reduces piracy, and potentially increases both monopoly profits and consumer surplus. Their work can be considered as complementary to our paper in that it provides an alternative mechanism through which an increase in IPRP can increase both firm profits and consumer surplus.

The remainder of the paper is organized in the following way: Section 2 sets up the basic model and provides a *short-run* analysis in which we investigate how the

[7]Our analysis, however, is much more general in that we use an arbitrary distribution of consumer types whereas he assumes a uniform distribution.

[8]Crampes and Laffont (2002) also analyze the effects of piracy on the pricing policy of a software producer. Their focus, however, is on the consequences of cost randomness in the decision for piracy and on the risk aversion of users.

monopolist's pricing decision is affected by the threat of piracy. We characterize the pattern of self-selection by consumers and the optimal price for the monopolist. In a comparative statics exercise, we show that the effects of increased copyright protection depend crucially on how it affects the two margins of the piracy costs discussed above. In Section 3, we extend the model to analyze the *long-run* implications of piracy for software quality. Section 4 contains concluding remarks.

2. The Model of Piracy: A Short-Run Analysis

Before analyzing the more complex effect of an increase in IPRP on software usage in the short run and development incentive in the long run, we first develop a simple model of piracy with a monopolistic software publisher for personal computers. There is a population of consumers whose total number is normalized to unity. Consumers are heterogeneous in their value of using the software. Let v denote a consumer's gross utility of using the software. The distribution of types is given by the *inverse* cumulative distribution function $F(v)$ with support $[0, \infty)$. It has a continuous density $F'(v) \leq 0$, that is, $F(v)$ denotes the proportion of consumers whose value of the software is *more* than v.

To analyze the *ex post* efficiency effects of piracy, we assume that the software is already developed and the marginal cost of production is zero. The copyright holder sets the price of the software p to maximize his revenues. As the consumer's utility v is private information, the copyright holder cannot price discriminate and charges a uniform price p.[9]

Optimal Pricing without Piracy: A Benchmark Case

As a benchmark case, we first consider a situation where the option to pirate copyrighted work is not available, that is, the consumers' only choice is whether to purchase or not.

[9] In a dynamic model, however, the monopolist can price discriminate consumers based on purchase history. See Fudenberg and Tirole (1998) for such an analysis.

The utility of buying an authorized copy is given by $U_B(v;p) = v - p$. We normalize the consumers' payoffs from not using the software to zero. Then, consumers whose valuation of software is more than p will purchase the software.

The purchase behavior of the consumers implies that the copyright holder maximizes his revenue:

$$\max_p \; R(p) = p \cdot F(p).$$

Since the monopolist's price p is uniquely determined by v, we will find it more convenient to treat v as the control variable:

$$\max_v \; R(v) = v \cdot F(v).$$

The marginal consumer v^* that maximizes the copyright holder's revenue is implicitly given by the first order condition:

$$F(v^*) + v^* \cdot F'(v^*) = 0. \tag{1}$$

We make the standard assumption that the distribution of types satisfies the monotone hazard rate condition, that is, $-F'/F$ is increasing:

$$-F''F + (F')^2 > 0. \tag{2}$$

This assumption ensures that the copyright holder's objective function is quasi-concave and the second order condition for the maximization problem is satisfied:

$$2 \cdot F'(v) + v \cdot F''(v) < 0.^{10} \tag{3}$$

Then, the number of software users is given by $F(v^*)$. The optimal price of the software for the copyright holder is $p^* = v^*$. Note that the optimal price and the marginal consumer without piracy depend only on the distribution of consumer types $F(\cdot)$.

Needless to say, the socially optimal price for the software, once it is developed, is its marginal cost, which is assumed to be zero. Due to monopolistic pricing,

[10] Using the first order condition, we can rewrite the second order condition as $2 \cdot F'(v) - F''(v) \cdot F(v) / F'(v) < 0$. The second order condition holds if the distribution F satisfies the monotone hazard rate condition. This condition is a standard assumption in the incentive literature and is satisfied by most widely used distributions; see Fudenberg and Tirole (1991, p. 267).

consumers whose types are below v* do not use the software and the deadweight loss is given by $-\int_0^{v^*} x dF(x)$.

Optimal Pricing with Piracy

Now we introduce the possibility of using the software through piracy without purchasing a legal copy. Piracy saves the price of the software for consumers. However, it entails potentially two types of costs. First, the unauthorized copy may not be a perfect substitute for the legal copy and typically entail some degree of quality degradation. We assume that this cost is proportional to the valuation of the consumer for the original, that is, the valuation of the type v consumer for the unauthorized copy is given by $(1-\alpha)\cdot v$, where α is the parameter for quality degradation.[11] In addition, we assume that illegal copying entails reproduction cost of *c*, which is assumed to be the same across users. Thus, the utility of using an unauthorized copy is given by $U_{UC}(v) = (1-\alpha)\cdot v - c$.[12]

In order to have a meaningful analysis of unauthorized copying, we restrict our attention to the parameter regions in which the piracy constraint is binding, that is,

$$c/(1-\alpha) < p^* = v^* \qquad (4),$$

where v^* is defined by (1). This condition is satisfied if the degree of quality degradation α and/or the cost of copying c are not too high.

When the piracy condition (4) is binding, the monopolist's problem is complicated by the following fact; some consumers are better off copying the software due to the heterogeneity in the copy costs, especially, when the price of the software is not sufficiently low. In response to piracy, the monopolist has two choices. One option is to *limit price* the software so that copying is not an attractive option. The other option

[11] Another interpretation is that α represents the enforcement efforts by the authority. If illegal copiers are caught with the probability of α, in which case the software is confiscated as punishment, the valuation of using an illegal copy would be given by (1-α).

[12] If we let $w = \alpha v + c$, then $U_{UC}(v) = v - w$. Hence, we can denote *w* as the gross copy cost.

is to price the software to sell only to the highest types and allow copying for intermediate types of customers.

Limit Pricing Regime without Piracy

With the piracy constraint binding $\left(c/(1-\alpha) < p^* = v^*\right)$, the limit price should satisfy the following incentive constraint to eliminate the incentives to copy:

$$U_B(v;p) = v - p \geq (1-\alpha)v - c = U_{UC}(v) \text{ for any v} \geq c/(1-\alpha) \qquad (5)$$

We observe that $U_B(v;p) - U_{UC}(v)$ is increasing in v, which implies that if the constraint above holds for v, then it also holds for any v' such that $v'>v$. Thus, all we need is that the inequality above be satisfied for $v = c/(1-\alpha)$. This in turn implies that the limit price and the marginal type are given by $p^L = v^L = c/(1-\alpha)$. Notice that the no piracy incentive constraint (5) is always binding under the assumption $c/(1-\alpha) < p^* = v^*$.

Lemma 1. When the piracy constraint is binding, the optimal limit price that prevents the incentive to copy is given by $p^L = c/(1-\alpha)$. In this case, the monopolist's revenue is given by $R = p^L F(p^L) = \frac{c}{1-\alpha} \cdot F\left(\frac{c}{1-\alpha}\right)$.

Copying Regime ($p > p^L = c/(1-\alpha)$)

If $p > p^L = c/(1-\alpha)$, the no piracy constraint is violated for some v's that are higher than but close to $v^L = c/(1-\alpha)$. Each consumer has two different choices for using the software, which incur two different types of cost. First, when a consumer buys a legal copy from the monopolist, he has to pay the price (p) and enjoys the full quality of the software. However, with choice of making an illegal copy, his cost will be the sum of degradation of quality that is proportional to his own valuation of the software and a constant reproduction cost. We assume at this time that the parameters of IPRP (α for the degradation rate and c for the reproduction cost) are fixed.

When consumers make their usage decision, they choose the one that yields the highest net utility. For a given price of a legal copy ($p > p^L = c/(1-\alpha)$) and the level of IPRP, consumers' optimal choices can be divided as follows:

$$\frac{p-c}{\alpha} \le v \qquad \text{purchase a legal copy}$$

$$\frac{c}{1-\alpha} \le v < \frac{p-c}{\alpha} \qquad \text{make an illegal copy}$$

$$v < \frac{c}{1-\alpha} \qquad \text{no use.}$$

Now the monopolist should take into account that potential consumers have another option to obtain the software. The monopolist, therefore, maximizes

$$Max\ p \cdot F\left(\frac{p-c}{\alpha}\right).$$

Once again, we treat the marginal consumer type $v = (p-c)/\alpha$ as the control variable:

$$Max\ (\alpha v + c)F(v).$$

The first order condition

$$(\alpha v + c)F'(v) + \alpha F(v) = 0 \tag{6}$$

determines the marginal type of consumer $\tilde{v}$, who is indifferent from purchasing an authorized copy from the monopolist and making an unauthorized copy.[13] Therefore, with the option of making an illegal copy, the self-selective choice of consumers is shown in Figure 1. We now can compare the monopolist's pricing behavior with and without piracy.

Proposition 1. With the possibility of piracy, the price of the software is lowered compared to that of monopoly without piracy, thereby inducing more demand for *legal* copies. Increase in usage of both legal and illegal copies under the copying regime brings higher ex post usage for the software.

Proof. Evaluate (6) at v^* which is the marginal consumer when no copying is feasible: $\alpha[F(v^*) + v^* F'(v^*)] + cF'(v^*) = cF'(v^*) < 0$. Hence, $v^* > \tilde{v}$.

[13] Variables under the copying regime are denoted by a tilde ($\tilde{v}$).

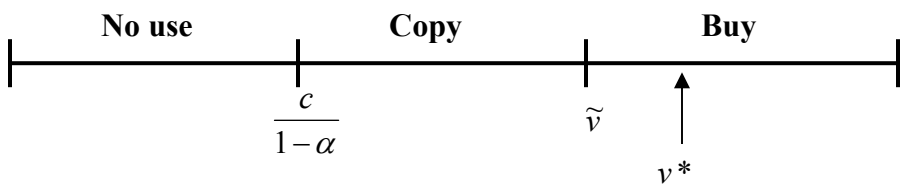


Figure 1. Consumers' Choice under Copying Regime

Under the copying regime, those consumers whose valuation lies between $c/(1-\alpha)$ and $\tilde{v}$ make illegal copies. Therefore, total ex post usage for the software is unambiguously increased with piracy as shown in Figure 1. By being just a threat (limit pricing regime) or an actual fact (copying regime), piracy has the same effect on the monopolist's pricing behavior: the price is lower than the one in the benchmark case. A more surprising result is that the usage of *legal* copies increases even in the presence of copying. Thus, the extent to which legal copies are used can be *complementary* with the extent of the usage of unauthorized copies, compared to the hypothetical case of no piracy regime.[14] The intuition for this result can be found in the monopolist's pricing behavior in response to the threat of unauthorized copying. If there were no price change, that is, at $p = p^* = v^*$ defined in (1), some of the previous purchasers of the legal copy will switch to the option of copying with the result of a lower number of legal copies being sold. Proposition 1, however, shows that the price reduction by the monopolist (from p^* to $\tilde{p} = \alpha\tilde{v} + c$) not only eliminates the incentives to switch for the previous buyers but also expands the base of buyers.

Comparative Statics

We now analyze the effects of marginal increase in the intellectual property rights. As with the previous studies in the literature (Novos and Waldman (1984), Yoon (2002), etc), we model the increase in IPRP as an increase in the cost of piracy ($w = \alpha v + c$, see footnote 12), which makes the option of piracy ($U_{UC}(v) = v - w$) less attractive. It is

[14] Our result does not assume network effects between authorized and unauthorized copies as in Slive and Bernhardt (1998) and Shy and Thisse (1999).

shown that the effects can have different implications depending on which regime the monopolist is operating under and the type of costs associated with piracy.

We only present the comparative statics results under the copying regime, since the marginal increases in IPRP from both the degradation rate and the reproduction cost have the same effect on the limit price of $p^L = c/(1-\alpha)$; as α or c increases, the monopolist has more market power and increases the limit price in line with one's intuition.

Proposition 2. Under the copying regime, as expected, the monopoly price increases with the strengthening of IPRP. The effects of an increase in IPRP on the authorized usage of software, however, are ambiguous depending on the types of costs associated with piracy. Higher degradation rate induces *less* authorized usage whereas higher reproduction cost induces *more* authorized usage [see Table 1].

Proof. Total differentiation of the first-order condition with respect to c and α respectively yields:

$$[2\alpha F'(\tilde{v}) + \alpha \tilde{v} F''(\tilde{v}) + cF''(v)]dv = -F'(\tilde{v})dc\,.$$

$$[2\alpha F'(\tilde{v}) + \alpha \tilde{v} F''(\tilde{v}) + cF''(v)]dv = -(F(\tilde{v}) + \hat{v}F'(\tilde{v}))d\alpha\,.$$

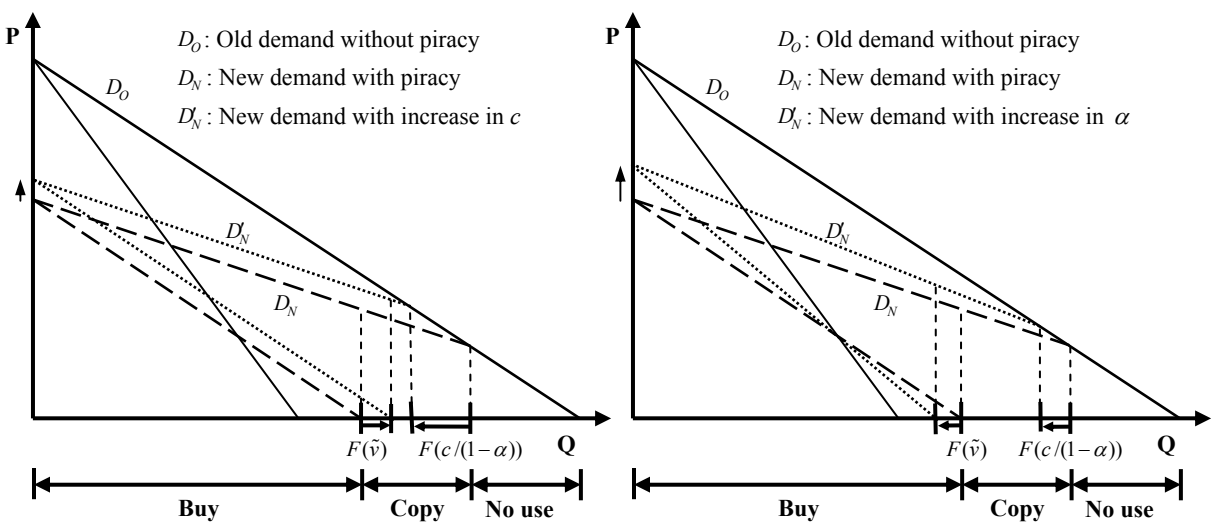
$$\frac{d\tilde{v}}{dc} = \frac{-F'(\tilde{v})}{|H|} < 0 \text{ and } \frac{d\tilde{v}}{d\alpha} = \frac{-(F(\tilde{v}) + \tilde{v}F'(\tilde{v}))}{|H|} = \frac{1}{|H|}\frac{cF'(\tilde{v})}{\alpha} > 0$$

where $|H| = 2\alpha F'(\tilde{v}) + \alpha \tilde{v} F''(\tilde{v}) + cF''(v) < 0$ by the second-order condition.

	Benchmark	**Limit Pricing**	**Copying Regime**
The monopolist's optimal choice	v^* p^*	v^L ($v^* > v^L$) p^L ($p^L < p^*$)	$\tilde{v}$ ($v^* > \tilde{v}$) $\tilde{p}$ ($\tilde{p} < p^*$)
An increase in the reproduction cost	*N/C*	$\frac{\partial v^L}{\partial c} > 0$ and $\frac{\partial p^L}{\partial c} > 0$	$\frac{\partial \tilde{v}}{\partial c} < 0$ and $\frac{\partial \tilde{p}}{\partial c} > 0$
An increase in the degradation rate	*N/C*	$\frac{\partial v^L}{\partial \alpha} > 0$ and $\frac{\partial p^L}{\partial \alpha} > 0$	$\frac{\partial \tilde{v}}{\partial \alpha} > 0$ and $\frac{\partial \tilde{p}}{\partial \alpha} > 0$

Table 1. Comparative Statics Results in the Short Run

With higher reproduction cost, all consumers face the same increase in the gross copy cost, which is equivalent to an outward *parallel* shift in demand for legal copies. With an increased demand, the monopolist responds with a price hike. The price increase, however, does not completely offset the initial demand increase with the result of increased sales. In contrast, if an increase in IPRP is derived from higher degradation rate, we observe a *pivot* change in demand that affects the *slope* of the demand curve for legal copies [see Figure 2]. Due to proportional increase in the gross copy cost, higher valuation consumers are more adversely affected by an increase in the degradation cost. A steeper demand curve means that elasticity of consumers is lower with more market power. Thus, the monopolist is more interested in serving only the high valuation consumers.



(a) The effect of reproduction cost increase (b) The effect of the degradation rate increase

Figure 2. The Effects of an Iincrease in IPRP with Linear Demand Case

Welfare Effects of Increase in IPRP in the Short Run

We are now in position to examine the effects of an increase in IPRP on social welfare. When the monopolist's optimal choice is limit pricing, it is straightforward to show the effect of an increase in IPRP on social welfare. As either the degradation or the

reproduction cost increases, making an illegal copy becomes less attractive. In response to this, the monopolist is able to charge a higher price and fewer consumers use a legal copy. The increased profit margin is only a monetary transfer from consumers to the monopolist. Social welfare is reduced as a result of less authorized usage.

If the monopolist's optimal choice is accommodation of piracy, the welfare effects of an increase in IPRP depend on the types of costs associated with piracy.

Proposition 3. Under the copying regime, the effects on social welfare of an increase in IPRP depend on the types of costs associated with piracy. Social welfare decreases with an increase in the degradation rate (α). However, the effects of an increase in the reproduction cost (c) on social welfare are ambiguous.

Proof. The social welfare can be derived from the sum of the monopolist's revenue and the consumer's surplus:

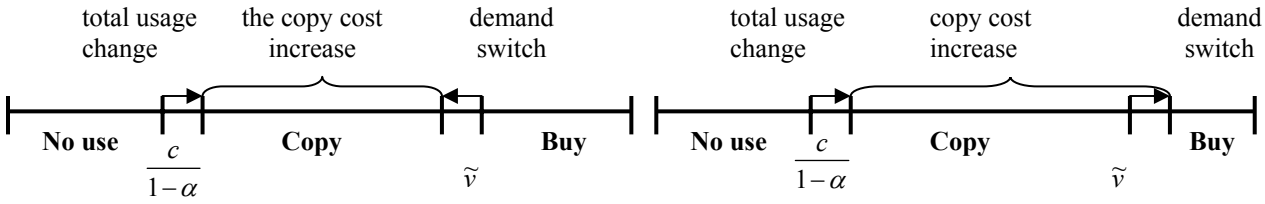
$$\begin{aligned} SW(\tilde{p}) &= R(\tilde{p}) + CS(\tilde{p}) \\ &= (\alpha\tilde{v} + c)F(\tilde{v}) - \int_{\tilde{v}}^{\infty} (v - \alpha\tilde{v} - c)F'(v)dv - \int_{\frac{c}{1-\alpha}}^{\tilde{v}} [(1-\alpha)v - c]F'(v)dv \\ &= \tilde{v}F(\tilde{v}) + \int_{\tilde{v}}^{\infty} F(v)dv - \int_{\frac{c}{1-\alpha}}^{\tilde{v}} [(1-\alpha)v - c]F'(v)dv . \end{aligned}$$

We examine the effect of an increase of IPRP on social welfare as

$$\frac{\partial SW(\tilde{p})}{\partial \alpha} = \underbrace{[\tilde{v} - ((1-\alpha)\tilde{v} - c)]F'(\tilde{v})\frac{\partial \tilde{v}}{\partial \alpha}}_{\text{demand switch}\,(-)} + \underbrace{\int_{\frac{c}{1-\alpha}}^{\tilde{v}} vF'(v)dv}_{\text{copy cost increase}\,(-)} < 0$$

$$\frac{\partial SW(\tilde{p})}{\partial c} = \underbrace{[\tilde{v} - ((1-\alpha)\tilde{v} - c)]F'(\tilde{v})\frac{\partial \tilde{v}}{\partial c}}_{\text{demand switch}\,(+)} + \underbrace{\int_{\frac{c}{1-\alpha}}^{\tilde{v}} F'(v)dv}_{\text{copy cost increase}\,(-)} \begin{matrix} < 0 \\ > 0 \end{matrix}$$

As can be seen from the expressions above, we can separate two different channels through which an increase in IPRP affects social welfare. The second term in each equation is always negative and represents social welfare loss due to increase in consumers gross copy cost caused by an increase in IPRP for consumers who continue to copy. The first term of each equation represents the demand switch effect between legal and illegal copies, which induces welfare gain or loss depending on the direction of demand switches. It decreases social welfare in case of an increase in the degradation rate (α), since the marginal consumers ($\tilde{v}$) who were indifferent between the legal and illegal copies now switch to illegal copies that are produced inefficiently and suffer from degradation. Taken together, both the demand switch effect and increased gross copy cost affect social welfare adversely with an increase in the degradation rate (α). In case of an increase in the reproduction cost (c), however, the demand switch effect is *positive* since it induces marginal consumers to switch to legal copies as we have demonstrated earlier. Therefore, the overall effect on social welfare is ambiguous and depends on the relative magnitude of the two countervailing effects [see Figure 3].[15]



(a) Welfare effect of higher reproduction (b) Welfare effect of higher degradation rate

Figure 3. Welfare Effect of the Two Margins of Piracy Costs

[15] As can be seen from Figure3, there is a third effect (total usage change) coming from marginal consumers $c/(1-\alpha)$ who are indifferent between copying and no consumption. However, these consumers have zero surplus and the effect on social surplus is of second-order and does not show up in the equations.

3. Copyright Protection and Development Incentives

Up to now, we have analyzed the effects of an increase in copyright protection on pricing and the incentives to pirate once the software has been produced. In this section, we analyze the long-term effects of an increase in IPRP on the incentive to create.[16] To analyze this issue, we introduce the cost of creating the software and endogenize the quality of the software. Let θ measure the quality of software that is created at cost $C(\theta)$ (with $C'(\theta) > 0,\ C''(\theta) > 0$). The quality of the good enters positively into the utility of consumers; the utility of consumer of type v is $\theta \cdot v$.

We consider now the long-term effects of piracy in which the monopolist decides not only on the pricing of the software but also on the quality of the software. We continue to assume that the marginal cost of software is fixed at zero regardless of the quality of the software once it has been developed.

Software Quality without Piracy: A Benchmark Case

Before analyzing the monopolist's quality provision with the possibility of piracy, we revisit the benchmark case, in which the consumers' only decision is whether to purchase or not. Since the consumers' payoff from not using the software is zero, consumers with non-negative net utilities purchase an official copy: $\theta \cdot v - p \geq 0$. Given the purchase decision of consumers, the copyright holder selects a price and a level of quality that maximize his profit:[17]

$$\max_{v,\theta}\ \pi \equiv \theta \cdot v \cdot F(v) - C(\theta).$$

The first order conditions

[16] If the monopoly power obtains *ex post* as a result of R&D competition, the analysis of software development should probably involve an oligopoly analysis. We thank an anonymous referee for pointing this one out. However, a full treatment of competition at the development stage is beyond the scope of this paper. One possible example for the monopolist's software development in a dynamic perspective is that the monopoly already introduced the copyrighted software and he decides the pricing and the quality of the upgrade. For an analysis of piracy in oligopoly, see Belleflamme and Picard (2005).

[17] Again, we use v as a control instead of p.

$$\frac{\partial \pi}{\partial v} = \theta \cdot F(v) + \theta \cdot v \cdot F'(v) = 0 \tag{7}$$

$$\frac{\partial \pi}{\partial \theta} = v \cdot F(v) - C'(\theta) = 0 \tag{8}$$

determine the marginal consumer $v*$ and the monopolist's optimal level of quality $\theta*$.

Proposition 4. Given the number of software users $F(v*)$, the quality of the software is sub-optimally low compared to the social optimum.

Proof. Given the number of software users $F(v*)$, i.e. all consumers of type $v \geq v*$ buying the software, the socially optimal quality of the software can be found by solving

$$\max_{\theta} \; -\int_{v*}^{\infty} \theta \cdot v \cdot F'(v)dv - C(\theta).$$

The first order condition

$$-\int_{v*}^{\infty} v \cdot F'(v)dv - C'(\theta^{opt}) = 0$$

determines the socially optimal level of quality θ^{opt}. Integration by parts of the first term on the left hand side shows $-\int_{v*}^{\infty} v \cdot F'(v)dv \geq v* \cdot F(v*)$. This implies that $C'(\theta^{opt}) \geq C'(\theta*)$ as we have $v* \cdot F(v*) = C'(\theta*)$ from (8). Therefore, the level of the software quality provided by the monopolist is sub-optimally low: $\theta^{opt} \geq \theta*$.

The intuition for this result is the following. The choice of $\theta*$ by the monopolist is determined by the marginal type $v*$. An increase in the benefit for the marginal consumer is captured via higher price of the software by the copyright holder. The effect on the inframarginal consumers is irrelevant for the monopolist as he cannot price discriminate among consumers. In contrast, the second-best level θ^{opt} is determined by the aggregate (or average) benefits for all consumers with values $[v*, \infty)$. As the *average* consumer's marginal valuation for the software quality is higher than the one for the

marginal consumer, the second-best level of quality of the software exceeds the one provided by the monopolist.[18]

Software Quality with Piracy

Now we turn to the monopolist's choice of the software quality when he faces piracy. To serve this purpose we extend the previous optimal pricing framework with the monopolist's choice of the software quality. The degradation rate now affects the valuation of the type v consumer for the unauthorized copy as $(1-\alpha)\cdot\theta\cdot v$. Thus, the utility of using an unauthorized copy is given by $U_{UC}(v) = (1-\alpha)\cdot\theta\cdot v - c$. We have the same restriction to the parameter regions, in which the piracy constraint is binding, that is,

$$c/(1-\alpha) < p^* = \theta^* v^* \qquad (9),$$

where θ^* and v^* are defined by (7) and (8). Therefore, with the binding constraint (9), the monopolist practices either limit pricing or accommodation to piracy.

Software Quality under the Limit Pricing Regime

With the piracy constraint (9) binding, the monopolist faces the following constrained profit maximization problem:

$$\underset{p^L,\theta}{Max}\ \pi^L = p^L F(\frac{p^L}{\theta}) - C(\theta)$$

Subject to

$$(1-\alpha)\theta v - c \le \theta v - p$$

Since the constraint is always binding under the assumption $c/(1-\alpha) < p^* = \theta^* v^*$, the monopolist determines the optimal choice of the software quality with the optimal limit price, $p^L = \frac{c}{1-\alpha}$, as following:

[18] This point is closely related to a monopolist's choice on product quality; see Spence (1975) and Tirole (1988, pp. 100-102).

$$\underset{\theta}{Max}\,\pi^L = \left(\frac{c}{1-\alpha}\right)\cdot F\left(\frac{c}{(1-\alpha)\theta}\right) - C(\theta).$$

The first order condition

$$\frac{\partial \pi^L}{\partial \theta} = -\frac{c^2}{(1-\alpha)^2\theta^2}F'\left(\frac{c}{(1-\alpha)\theta}\right) - C'(\theta) = 0 \tag{10}$$

determines the monopolist's optimal choice of software quality.

Proposition 5. Under the limit pricing regime, the monopolist chooses a lower level of quality than that of monopoly without piracy.

Proof. Let us evaluate the first order condition (10) at θ* which is the level of quality without piracy.

$$\left.\frac{\partial \pi^L}{\partial \theta}\right|_{\theta^*} = -\frac{c^2}{(1-\alpha)^2(\theta^*)^2}F'\left(\frac{c}{(1-\alpha)\theta^*}\right) - C'(\theta^*)$$

We know that

$$-\frac{c^2}{(1-\alpha)^2(\theta^*)^2}\cdot F'\left(\frac{c}{(1-\alpha)\theta^*}\right) < \frac{c}{(1-\alpha)\ \theta^*}\cdot F\left(\frac{c}{(1-\alpha)\theta^*}\right) < v^* F(v^*)$$

The inequalities above follow from the fact that $vF(v)$ is a concave function which is maximized at v^* and our assumption that $\frac{c}{1-\alpha} < p^* = \theta^* v^*$. Thus, we have $\left.\frac{\partial \pi^L}{\partial \theta}\right|_{\theta^*} < v^* F(v^*) - C'(\theta^*) = 0$, which implies that $\theta^L < \theta^*$.

Software Quality under the Copying Regime

Under the copying regime consumers compare the payoffs from buying an authorized copy [$\theta \cdot v - p$] or making an unauthorized copy [$(1-\alpha)\cdot\theta\cdot v - c$]. Given the purchase behavior of consumers, the monopolist maximizes his profit:

$$\max_{v,\theta}\ \tilde{\pi} = (\alpha\cdot\theta\cdot v + c)F(v) - C(\theta).$$

The first order conditions

$$\frac{\partial \tilde{\pi}}{\partial v} = (\alpha\theta\ v + c)F'(v) + \alpha\theta F(v) = 0 \tag{11}$$

$$\frac{\partial \tilde{\pi}}{\partial \theta} = \alpha v F(v) - C'(\theta) = 0 \tag{12}$$

again determine the marginal consumer $\tilde{v}$ and the software quality $\tilde{\theta}$.

Proposition 6. Under the copying regime the existence of piracy leads to a further underprovision of the software quality compared to that of monopoly without piracy but more authorized usage.

Proof. Evaluating (11) at $v = v^*$ yields $(\alpha\theta\, v^* + c)F'(v^*) + \alpha\theta\, F(v^*) = cF'(v^*) < 0$. Hence we have $\tilde{v} < v^*$. Also, evaluating (12) at $v = v^*$ yields $-v^* F(v^*)(1-\alpha) < 0$ and therefore $\tilde{\theta} < \theta^*$.

The intuition underlying this result is the following. The choice of $\tilde{\theta}$ by the monopolist is determined by the marginal type $\tilde{v}$. At the development stage of the software the monopolist expects that the marginal consumer is not v^* but $\tilde{v}$ when there is piracy. Given this anticipation, the optimal choice of the software quality should be lower than the one from the benchmark case. As seen from Proposition 6, the software quality provision by the monopolist is already sub-optimally low ($\theta^* < \theta^{opt}$) even without the threat of piracy. The existence of piracy aggravates this inefficient provision of the software quality in both regimes. Our result thus lends theoretical support for the claim that piracy reduces the incentives to develop new software.[19]

Comparative Statics

We now turn to analysis of the effects of marginal increase in IPRP on the monopolist's development incentive. In the limit pricing regime, the monopolist lowers his price until the constraint $(1-\alpha)\theta v - c \le \theta v - p$ is binding to eliminate piracy. The maximum price he can charge under limit pricing is $p^L = c/(1-\alpha)$, which depends on the relative level

[19] We assume that for each user, the ratio between the gross utilities of using an unauthorized copy and the original product is left unchanged when quality is improved. One response to piracy by the monopolist can be concentrating on improvements in software quality primarily targeted toward the very features that make an authorized copy more valuable than an unauthorized one. In such a case, the monopolist may have higher incentives to enhance quality, but there could be distortions in the types of quality improvements in that they are not necessarily the ones most valued by consumers.

of the degradation rate (α) and the reproduction cost (c). Increases in IPRP from either the degradation rate or the reproduction cost induce less authorized usage, which is equivalent to higher valuation from the marginal consumer v^L. This induces the monopolist to provide a higher quality.

Proposition 7. Under the copying regime the effects of increase in IPRP depend on the types of costs associated with piracy. Higher degradation rate induces higher quality and less legal usage. In contrast, higher reproduction cost results in lower quality and more authorized usage.

Proof. See the Appendix.

	Benchmark	**Limit Pricing**	**Copying Regime**
The monopolist's optimal choice	v^* p^* θ^*	v^L $(v^* > v^L)$ p^L $(p^L < p^*)$ θ^L $(\theta^L < \theta^*)$	$\tilde{v}$ $(v^* > \tilde{v})$ $\tilde{p}$ $(\tilde{p} < p^*)$ $\tilde{\theta}$ $(\tilde{\theta} < \theta^*)$
An increase in the reproduction cost	*N/C*	$\frac{\partial v^L}{\partial c} > 0$ and $\frac{d\theta^L}{dc} > 0$	$\frac{\partial \tilde{v}}{\partial c} < 0$ and $\frac{d\tilde{\theta}}{dc} < 0$
An increase in the degradation rate	*N/C*	$\frac{\partial v^L}{\partial \alpha} > 0$ and $\frac{d\theta^L}{d\alpha} > 0$	$\frac{\partial \tilde{v}}{\partial \alpha} > 0$ and $\frac{d\tilde{\theta}}{d\alpha} > 0$

Table 2. Comparative Statics Results in the Long Run

Table 2 summarizes our results. Since the monopolist's quality provision is determined by the marginal consumer's valuation for the software, the effects of increase in IPRP depend on the change of the marginal consumer, which is different according to types of costs associated with piracy. With higher reproduction cost, all consumers face the same increase in the gross copy cost, which is equivalent to overall demand increase for the monopolist. Hence, the monopolist benefits from higher demand by charging a higher price, yet increasing sales at the same time. Facing the marginal consumer's lower valuation, the monopolist has less incentive to provide higher quality. In contrast, if an

increase in IPRP is derived from higher degradation rate, we observe proportional increase in the gross copy cost and comparatively more market power for the monopolist. With increase in market power, the monopolist charges a higher price by focusing on high valuation consumers. Responding to the marginal consumer's higher valuation, the monopolist has more incentive to supply higher quality.

In terms of social welfare, we have more traditional results when the increase in IPRP is due to higher degradation rate; there is a trade-off between short-run (usage) welfare and long-run (development) welfare since it induces higher quality but less usage. When the increase in IPRP is in the form of higher reproduction cost, however, the trade-off between static and dynamic efficiency may not exist. In particular, in the region where static welfare decreases with an increase in c, the increase in IPRP through higher reproduction cost unambiguously reduces social welfare since it also reduces the level of quality provision by the monopolist.

The result in Proposition 7 has a similar flavor to the one derived by Valletti and Szymanski (forthcoming) in their analysis of parallel trade. In particular, they conduct a welfare analysis of parallel trade for products protected by intellectual property rights such as pharmaceuticals. When competition from a "generic" is present, Valleti and Szymanski find that *ex post* the generic extends the circumstances under which parallel import is welfare enhancing. However, the presence of the generic exacerbates the *ex ante* incentives to invest and the quality reduction due to parallel trade is magnified under parallel trade. Even though they address different types of questions, a generic pharmaceutical product in their model plays a similar role to a pirated copy in our model.

4. Concluding Remarks

In this paper, we develop a simple model of piracy to analyze implications of increased intellectual property rights on the short-run and long-run resource allocations. In a model of self-selection with heterogeneous users, we show that the consumers'

option to use illegal copies constrains the copyright holder's ability to charge a monopoly price. Consequently, the possibility of piracy leads to more usage of legal copies. In this sense, the presence of unauthorized copies acts as a complement to the usage of legal copies rather than a substitute, compared to the benchmark case of no piracy.

To analyze the effects of an increase in IPRP more precisely, we consider two types of costs associated with piracy; the type-independent *reproduction cost* and the type-dependent *degradation cost*. We provide a theoretical framework to show that the effects of piracy depend crucially on the nature of piracy costs. In particular, strengthening IPRP in the form of an increase in the degradation cost supports the conventional wisdom on IPRP. It reduces social welfare in the short-run by providing the monopolist with more market power, which results in both negative demand switch and total usage change. In the long-run, the monopolist facing a higher marginal consumer type has more incentive to provide higher quality. Thus, there is a trade-off between short-run and long-run efficiency. In contrast, an increase in the reproduction cost induces more authorized usage of the software in the short-run. Even though an increase in IPRP with higher reproduction cost reduces the total usage of the software, more consumers obtain the software from the monopolist with more efficient technology. Therefore, an increase in the reproduction cost may increase or decrease social welfare in the short run. Moreover, due to the marginal consumer's lower valuation for the software, the monopolist has less incentive to provide higher quality in the long-run. Thus, we cannot rule out the case where an increase IPRP reduces social welfare both in the short-run and long-run. The results in the paper thus suggest that any policy implementation of IPRP should pay more attention to how the policy change will affect the two margins of piracy costs, not just the overall piracy costs.

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Appendix : Proof of Proposition 7

By totally differentiating (11) and (12), we have

$$\begin{bmatrix} \dfrac{\partial^2 \pi}{\partial v^2} & \dfrac{\partial^2 \pi}{\partial v \partial \theta} \\ \\ \dfrac{\partial^2 \pi}{\partial \theta \partial v} & \dfrac{\partial^2 \pi}{\partial \theta^2} \end{bmatrix} \begin{bmatrix} \dfrac{dv}{dc} \\ \\ \dfrac{d\theta}{dc} \end{bmatrix} = \begin{bmatrix} -\dfrac{\partial^2 \pi}{\partial v \partial c} \\ \\ -\dfrac{\partial^2 \pi}{\partial \theta \partial c} \end{bmatrix}.$$

By using Cramer's rule, we have

$$\frac{dv}{dc} = \frac{1}{|H|} \begin{vmatrix} -\dfrac{\partial^2 \pi}{\partial v \partial c} & \dfrac{\partial^2 \pi}{\partial v \partial \theta} \\ \\ -\dfrac{\partial^2 \pi}{\partial \theta \partial c} & \dfrac{\partial^2 \pi}{\partial \theta^2} \end{vmatrix}, \text{ and } \frac{d\theta}{dc} = \frac{1}{|H|} \begin{vmatrix} \dfrac{\partial^2 \pi}{\partial v^2} & -\dfrac{\partial^2 \pi}{\partial v \partial c} \\ \\ \dfrac{\partial^2 \pi}{\partial \theta \partial v} & -\dfrac{\partial^2 \pi}{\partial \theta \partial c} \end{vmatrix}.$$

where $|H| = \dfrac{\partial^2 \pi}{\partial v^2} \cdot \dfrac{\partial^2 \pi}{\partial \theta^2} - \left(\dfrac{\partial^2 \pi}{\partial v \partial \theta} \right)^2$ is the determinant of the Hessian matrix with $|H| > 0$

by the second-order condition for maximization.

$$\frac{dv}{dc} = \frac{1}{|H|}(F'(v)C''(\theta)) < 0.$$ [20]

$$\frac{d\theta}{dc} = \frac{1}{|H|}\left(-\frac{c(F'(v))^2}{\theta} \right) < 0.$$ [21]

By totally differentiating the first-order conditions, we have

$$\begin{bmatrix} \dfrac{\partial^2 \pi}{\partial v^2} & \dfrac{\partial^2 \pi}{\partial v \partial \theta} \\ \\ \dfrac{\partial^2 \pi}{\partial \theta \partial v} & \dfrac{\partial^2 \pi}{\partial \theta^2} \end{bmatrix} \begin{bmatrix} \dfrac{dv}{d\alpha} \\ \\ \dfrac{d\theta}{d\alpha} \end{bmatrix} = \begin{bmatrix} -\dfrac{\partial^2 \pi}{\partial v \partial \alpha} \\ \\ -\dfrac{\partial^2 \pi}{\partial \theta \partial \alpha} \end{bmatrix}.$$

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[20] It can be easily verified that $\dfrac{\partial^2 \pi}{\partial v \partial c} = F'(v) < 0$, $\dfrac{\partial^2 \pi}{\partial \theta^2} = -C''(\theta) < 0$, $\dfrac{\partial^2 \pi}{\partial \theta \partial c} = 0$, and

$\dfrac{\partial^2 \pi}{\partial v \partial \theta} = \alpha v F'(v) + \alpha F(v) = -\dfrac{cF'(v)}{\theta} > 0$.

[21] It can be also easily verified that, $\dfrac{\partial^2 \pi}{\partial v^2} < 0$, $\dfrac{\partial^2 \pi}{\partial \theta \partial c} = 0$, $\dfrac{\partial^2 \pi}{\partial v \partial c} = F'(v) < 0$, and

$\dfrac{\partial^2 \pi}{\partial \theta \partial v} = \alpha v F'(v) + \alpha F(v) = -\dfrac{cF'(v)}{\theta} > 0$.

By using Cramer's rule, we have

$$\frac{dv}{d\alpha} = \frac{1}{|H|}\begin{vmatrix} -\dfrac{\partial^2\pi}{\partial v\partial\alpha} & \dfrac{\partial^2\pi}{\partial v\partial\theta} \\ -\dfrac{\partial^2\pi}{\partial\theta\,\partial\alpha} & \dfrac{\partial^2\pi}{\partial\theta^{\,2}} \end{vmatrix}, \text{ and } \frac{d\theta}{d\alpha} = \frac{1}{|H|}\begin{vmatrix} \dfrac{\partial^2\pi}{\partial v^2} & -\dfrac{\partial^2\pi}{\partial v\partial\alpha} \\ \dfrac{\partial^2\pi}{\partial\theta\,\partial v} & -\dfrac{\partial^2\pi}{\partial\theta\,\partial\alpha} \end{vmatrix}.$$

$$\frac{dv}{d\alpha} = -\frac{1}{|H|}F'(v)\left[\frac{c}{\alpha}C''(\theta) + \frac{cv}{\theta}F(v)\right] > 0\,.$$ [22]

$$\frac{d\theta}{d\alpha} = \frac{1}{|H|}\left[\frac{\partial^2\pi}{\partial v^2}(-vF(v)) + \frac{c^2}{\alpha^{\,2}\theta}\{F'(v)\}^2\right] > 0\,.$$ [23]

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[22] We have $\dfrac{\partial^2\pi}{\partial v\partial\alpha} = \dfrac{-cF'(v)}{\alpha} > 0$, $\dfrac{\partial^2\pi}{\partial\theta^{\,2}} = -C''(\theta) < 0$ $\dfrac{\partial^2\pi}{\partial\theta\,\partial\alpha} = vF(v) > 0$, and $\dfrac{\partial^2\pi}{\partial v\partial\theta} = \alpha\, vF'(v) + \alpha\, F(v) = -\dfrac{cF'(v)}{\theta} > 0$.

[23] We have $\dfrac{\partial^2\pi}{\partial v^2} < 0$, $\dfrac{\partial^2\pi}{\partial\theta\,\partial\alpha} = vF(v) > 0$, $\dfrac{\partial^2\pi}{\partial v\partial\alpha} = \dfrac{-cF'(v)}{\alpha} > 0$, and $\dfrac{\partial^2\pi}{\partial\theta\,\partial v} = vF'(v) + F(v) = -\dfrac{cF'(v)}{\alpha\,\theta} > 0$.