Math 105 History of Mathematics

First Test

Prof. D. Joyce, October, 2010

You may refer to one sheet of notes on this test. Points for each problem are in square brackets. Start your answers to each problem on a separate page page of the bluebook. Please write or print clearly.

Problem 1. Essay. [25] Select one of the three topics A, B, and C.

Please think about these topics and make an outline before you begin writing. You will be graded on how well you present your ideas as well as your ideas themselves. Each essay should be relatively short—one to three written pages. There should be no fluff in your essays. Make your essays well-structured and your points as clearly as you can.

Topic A. Explain the logical structure of the *Elements* (axioms, propositions, proofs). How does this differ from earlier mathematics of Egypt and Babylonia? How can such a logical structure affect the mathematical advances of a civilization?

Topic B. Compare and contrast the arithmetic of the Babylonians with that of the Egyptians. Be sure to mention their numerals, algorithms for the arithmetic operations, and fractions. Illustrate with examples. Don't go into their algebra or geometry for this essay.

Topic C. Aristotle presented four of Zeno's paradoxes: the Dichotomy, the Achilles, the Arrow, and the Stadium. Select one, but only one, of them and write about it. State the paradox as clearly and completely as you can. Explain why it was considered important. Refute the paradox, either as Aristotle did, or as you would from a modern point of view.

Problem 2. [15] Find the greatest common divisor of the two numbers 11484 and 7902 by using the Euclidean algorithm. (Computations are sufficient, but show your work. An explanation is not necessary.)

Problem 3. [24; 6 points each part] On Eudoxus' definition of equality of ratios of magnitudes. Answer each part in a couple of sentences.

a. The Pythagorean philosophy of numbers was summarized by the Pythagorean Philolaus as

> All things which can be known have number; for it is not possible that without number anything can be either conceived or known.

Explain why the discovery of incommensurable magnitudes (such as the side of a square and its diagonal) led to a crisis for the Pythagoreans.

b. Explain in your own words Eudoxus definition of equality of ratios, that is, when is the ratio $a : b$ of two magnitudes of one type equal to the ratio $c : d$ of two magnitudes of (possibly) another type.

c. Using this definition, show that the numeric ratio 3:5 does not equal the ratio 4:6.

d. Explain how this definition resolved the crisis and supported the Pythagorean philosophy.

Problem 4. [16] On areas of circles. The cultures we have studied—Egyptian, Babylonian, and Greek—all knew how to approximate the area of a circle. Choose one of the cultures and describe one method that was used to compute the area of a circle. Your description should only be a sentence or two long. Illustrate the method by detemining the area of a circle whose diameter is 9 cubits. (A cubit being a measure of length, the length of a forearm, used by all three cultures.)

Problem 5. [20; 5 points each part] True/false. For each sentence write the whole word "true" or the whole word "false". If it's not clear whether it should be considered true or false, you may explain in a sentence if you prefer.

a. Euclid's parallel postulate (Postulate 5 in Book I of the *Elements*) stated that lines in the same direction are parallel.

b. The ancient Babylonians knew the Pythagorean theorem (the square on the hypenenuse of a right triangle is equal to the sum of the squares on the other two sides) over a thousand years before Pythagoras.

c. Each of the propositions in Euclid's *Elements* includes a proof.

d. A triangular number is the perimeter of an equilateral triangle, for example, 15 is a triangular number since an equilateral triangle of side length 5 has perimeter 15.

e. Whereas Egyptians used common fractions like $\frac{2}{5}$, Babylonians prefered unit fractions like one-third plus one-fifteenth.