# Math 105 History of Mathematics

Second Test

Prof. D. Joyce, November, 2010

You may refer to one sheet of notes on this test, and you may use a calculator if you like. Points for each problem are in square brackets. Start your answers to each problem on a separate page page of the bluebook. Please write or print clearly.

**Problem 1.** Essay. [25] Select one of the two topics A or B.

Please think about these topics and make an outline before you begin writing. You will be graded on how well you present your ideas as well as your ideas themselves. Each essay should be relatively short—one to three written pages. There should be no fluff in your essays. Make your essays well-structured and your points as clearly as you can.

**Topic A.** One quality of the mathematics of ancient India and China, on the one hand, which differs from that of Greek and Islamic/Arabic mathematicians, on the other hand, is formalism. Here formalism means careful definitions and clear proofs. (Another aspect of formalism which doesn't have much role here is symbolism, as in symbolic algebra which wasn't developed until later.) Explain what this difference is and describe the role of formalism in the development of mathematics in those cultures.

**Topic B.** Trigonometry was part of the mathematical knowledge in several regions: Greece, India, China, and Islamic/Arabic. Briefly describe the trigonometry of each in a sentence or two. Describe how knowledge of trigonometry passed among these cultures.

**Problem 2.** [21; 7 points each part] On the Chinese algorithm for solving polynomial equations.

In this problem, you will use the Chinese algorithm for solving the cubic equation $x^3 + 2x^2 + 6x - 13258 = 0$.

Here is the first stage of the computation.

| | $x^3$ | $x^2$ | $x^1$ | $x^0$ |
|---|---|---|---|---|
| | 1 | 2 | 6 | $-13258$ |
| 20 | | 20 | 440 | 8920 |
| | 1 | 22 | 446 | $-4338$ |
| 20 | | 20 | 840 | |
| | 1 | 42 | 1286 | |
| 20 | | 20 | | |
| | 1 | 62 | | |

**a.** This computation shows that the first digit of a solution $x$ of the cubic equation is 2, that is to say, a solution lies between 20 and 30. If you try 30 instead of 20, the computations show that 30 is too large. What happens in those computations that tells you that 30 is too large?

**b.** The next digit turns out to be 2 (so that the solution lies between 22 and 23). Starting with the line

$$1 \qquad 62 \qquad 1286 \qquad -4338$$

perform the calculations for 2.

**c.** Now that we know the solution is between 22 and 23, estimate the next digit. You don't have to continue to use the algorithm; although it works, it's very time consuming. Rather, give an extimate and explain in a sentence how you arrived at that estimate. (This is historically accurate because when the numbers become small, another method was used to finish up.)

**Problem 3.** [12] The hundred fowls problem. This famous problem is stated as follows. Roosters cost 5 coins each. Hens cost 3 coins each. Chicks are three for 1 coin. If 100 fowls are bought for exactly 100 coins, then how many of the three (roosters, hens, and chicks) are bought?

**a.** [12] This specific problem was known in China, India, and Europe. It is likely that it was invented once and passed to other regions. If so, does that say anything about the passage of mathematics from one region to another? Explain.

**b.** [8 extra credit] Find a solution to the 100 fowls problem. You may use modern algebra if you like. For a valid solution, only whole numbers of creatures are allowed, and at least one of each of the three kinds is required; zero and negative numbers aren't allowed.

(Hints: There are three different solutions. The number of roosters has to be a multiple of 4.)

**Problem 4.** [18; 9 points each part] On Eratosthenes determination of the size of the earth.

Eratosthenes of Cyrene (274-194 BCE) is credited with measuring the earth. He found that at noon on the summer solstice the sun was directly overhead at Syene, while at the same time at Alexandria, approximately 500 stades due north, the sun was $\frac{1}{50}$ of a circle ($7\frac{1}{5}^{\circ}$) from the zenith (directly overhead).

**a.** Outline the argument that Eratosthenes used to determine the circumference of the earth.

**b.** List the assumptions that this argument is based on.

**Problem 5.** [24; 4 points each part] True/false. For each sentence write the whole word "true" or the whole word "false". If it's not clear whether it should be considered true or false, you may explain in a sentence if you prefer.

**a.** Combinations, that is, the number of ways of choosing $k$ things out of a set of $n$ things, were studied in ancient Greece, but were unknown in China and India until modern times.

**b.** Ptolemy's believed that the planets travelled around the Sun in ellipses as opposed to earlier astronomers who believed that they travelled around the earth in curves called epicycles.

**c.** Al-Khwarizmi is known for his early algebra, methods for solving equations.

**d.** Diophantus was a later Greek mathematician who is known primarily for his formula for the area of a triangle, $A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s$ is the semiperimeter $s = \frac{1}{2}(a+b+c)$.

**e.** Archemedes proved that two objects balance when their distances from a fulcrum are inversely proportional to their weights.

**f.** In ancient China, rods (rod numerals) were used to solve simulataneous systems of linear equations.