Math 105 History of Mathematics

Second Test Answers

Prof. D. Joyce, November, 2010

Scale. 85–104 A. 74–84 B. 59–73 C. Median 79.

Problem 1. Essay. [25] Select one of the two topics A or B.

Topic A. One quality of the mathematics of ancient India and China, on the one hand, which differs from that of Greek and Islamic/Arabic mathematicians, on the other hand, is formalism. Here formalism means careful definitions and clear proofs. (Another aspect of formalism which doesn't have much role here is symbolism, as in symbolic algebra which wasn't developed until later.) Explain what this difference is and describe the role of formalism in the development of mathematics in those cultures.

Points to make: The Greek mathematicians developed formalism. The first work that we have which exhibits formalism is Euclid's *Elements*. The Islamic/Arabic mathematicians learned of it from Greek works and included formalism in many of their mathematical works. The aspects of formalism are rarely found in India and China; definitions and proofs are very rarely included there. Instead, methods are described with examples.

Because Greek and Islamic/Arabic mathematics was based on formalism, there are a lot of theoretical developments. In India and China there are primarily advanced computational developments.

Topic B. Trigonometry was part of the mathematical knowledge in several regions: Greece, India, China, and Islamic/Arabic. Briefly describe the trigonometry of each in a sentence or two. Describe how knowledge of trigonometry passed among these cultures.

Points to make: Trigonometry developed first in Greek mathematics. The trigonometric function was the chord of an angle, the length of a certain chord in a large radius circle. It was used primarily in astronomy. In the early period trig passed to India where the sine function replaced the chord. From India it passed to China. Islamic/Arabic mathematics inherited trig from both Greek and Indian mathematics. Tangents also appeared in Chinese and Islamic/Arabic mathematics.

Problem 2. [21; 7 points each part] On the Chinese algorithm for solving polynomial equations.

In this problem, you will use the Chinese algorithm for solving the cubic equation $x^3 + 2x^2 + 6x - 13258 = 0$.

Here is the first stage of the computation.

	x^3	x^2	x^1	x^0
	1	2	6	-13258
20		20	440	8920
	1	22	446	-4338
20		20	840	
	1	42	1286	
20		20		
	1	62		

a. This computation shows that the first digit of a solution x of the cubic equation is 2, that is to say, a solution lies between 20 and 30. If you try 30 instead of 20, the computations show that 30 is too large. What happens in those computations that tells you that 30 is too large?

When you try 30, here's what the first line looks like

	x^3	x^2	x^1	x^0
	1	2	6	-13258
30		30	960	28980
	1	32	966	15722

Since the last entry is some positive number, that means 30 is too large.

b. The next digit turns out to be 2 (so that the solution lies between 22 and 23). Starting with the line

$$1 \qquad 62 \qquad 1286 \qquad -4338$$

perform the calculations for 2.

Here is the second stage of the computation.

	y^3	y^2	y^1	y^0
	1	62	1286	-4338
2		2	128	2828
	1	64	1414	-1510
2		2	132	
	1	66	1546	
2		2		
	1	68		

c. Now that we know the solution is between 22 and 23, estimate the next digit. You don't have to continue to use the algorithm; although it works, it's very time consuming. Rather, give an extimate and explain in a sentence how you arrived at that estimate. (This is historically accurate because when the numbers become small, another method was used to finish up.)

The last line corresponds to the equation $z^3 + 68z^2 + 1546z - 1510 = 0$. The solution z is less than 1 so both z^3 and z^2 are small. Therefore, the equation is close to the equation $1546z - 1510 = 0$, so z is about $\frac{1510}{1546}$, or about 0.98. So the next digit is 9. Thus, our answer so far is 22.9.

Problem 3. [12] The hundred fowls problem. This famous problem is stated as follows. Roosters cost 5 coins each. Hens cost 3 coins each. Chicks are three for 1 coin. If 100 fowls are bought for exactly 100 coins, then how many of the three (roosters, hens, and chicks) are bought?

a. [12] This specific problem was known in China, India, and Europe. It is likely that it was invented once and passed to other regions. If so, does that say anything about the passage of mathematics from one region to another? Explain.

It means that at least some mathematics was passed from one region to another, but we can't tell from this information which region was the source of the problem. Perhaps it even came from some other region originally (possibly Egypt or Babylonia, who knows?). This is a recreational mathematics problem, so it says at most that recreational problems were passed among the regions, and we can't tell from it whether more serious mathematics passed among regions.

b. [8 extra credit] Find a solution to the 100 fowls problem. You may use modern algebra if you like. For a valid solution, only whole numbers of creatures are allowed, and at least one of each of the three kinds is required; zero and negative numbers aren't allowed.

(Hints: There are three different solutions. The number of roosters has to be a multiple of 4.)

It's probably easiest if we treat groups of three chicks as a unit. Let one unknown, x, be the number of roosters, y the number of hens, and z the number of groups of three chicks. Then we get two equations

$$\begin{array}{rcrcrcl} x & + & y & + & 3z & = & 100 \\ 5x & + & 3y & + & z & = & 100 \end{array}$$

We're looking for solutions to these system of two linear equations in three unknowns which are all positive integers.

There are many ways to solve this system. Here's one where we eliminate z from the equations. From the second equation, $z = 100 - 5x - 3y$. Putting that value in the first equation and simplifying, we get

$$7x + 4y = 100.$$

Note that x has to be evenly divisible by 4, so set $x = 4w$. Then

$$7w + y = 25.$$

That gives us three possible values for w, namely, 1, 2, or 3. From those three values we get the three solutions

w	$x = 4w$ roosters	y hens	z	$3z$ chicks
1	4	18	26	78
2	8	11	27	81
3	12	7	28	84

Problem 4. [18; 9 points each part] On Eratosthenes' determination of the size of the earth.

Eratosthenes of Cyrene (274-194 BCE) is credited with measuring the earth. He found that at noon on the summer solstice the sun was directly overhead at Syene, while at the same time at Alexandria, approximately 5000 stades due north, the sun was $\frac{1}{50}$ of a circle ($7\frac{1}{5}^\circ$) from the zenith (directly overhead).

a. Outline the argument that Eratosthenes used to determine the circumference of the earth.

The angle at Alexandria between the sun and the zenith is $\frac{1}{50}$ of a circle. That's equal to the angle at the center of the earth between Alexandria and Syene. Therefore, the arc of the circumference between Alexandria and Syene is $\frac{1}{50}$ of the whole circumference. Since that arc is 5000 stades, therefore the whole circumference is 50 times that, namely, 250,000 stades. (That's fairly close to the actual circumference, but about 15% high.)

b. List the assumptions that this argument is based on.

There are two major assumptions. First, the earth is a sphere. Second, the sun is infinitely far away. There are various minor assumptions.

Problem 5. [24; 4 points each part] True/false.

a. Combinations, that is, the number of ways of choosing k things out of a set of n things, were studied in ancient Greece, but were unknown in China and India until modern times.

False on all counts.

b. Ptolemy's believed that the planets travelled around the Sun in ellipses as opposed to earlier astronomers who believed that they travelled around the earth in curves called epicycles.

False. He used epicycles around the earth. It wasn't until much later (Kepler) that ellipses around the sun were used.

c. Al-Khwarizmi is known for his early algebra, methods for solving equations.

True.

d. Diophantus was a later Greek mathematician who is known primarily for his formula for the area of a triangle, $A = \sqrt{s(s-a)(s-b)(s-c)}$ where s is the semiperimeter $s = \frac{1}{2}(a+b+c)$.

False, that's Heron. Diophantus did number theory.

e. Archemedes proved that two objects balance when their distances from a fulcrum are inversely proportional to their weights.

True. He proved the law of the lever.

f. In ancient China, rods (rod numerals) were used to solve simulataneous systems of linear equations.

True.