

Math 105 History of Mathematics

First Test

Prof. D. Joyce, March, 2013

You may refer to one sheet of notes on this test. Points for each problem are in square brackets. You may do the problems in any order that you like, but please start your answers to each problem on a separate page of the bluebook. Please write or print clearly.

Problem 1. Essay. [30] Select *one* of the three topics A, B, and C.

Please think about these topics and make an outline before you begin writing. You will be graded on how well you present your ideas as well as your ideas themselves. Each essay should be relatively short—one to three written pages. There should be no fluff in your essays. Make your essays well-structured and your points as clearly as you can.

Topic A. Contrast the level of Greek mathematics of the 4th century BC (the time of Eudoxus, Plato, Theaetetus, and ending with Euclid) with that of the Egyptians and Babylonians of the second millennium (the time of the Ahmes papyrus & Plimpton 322 tablet). Rather than trying to compare all of mathematics, choose only one main subject for comparison. Some example subjects: the Pythagorean theorem, formalism, numbers vs. geometry, algebra, similar figures, areas of circles.

Topic B. We've discussed mathematics of Egypt, Babylonia, and Greece. Briefly summarize their transmission between cultures. Identify mathematics that may have been transmitted from one culture to one of the others. Explain why you think it may have been transmitted.

Topic C. On Babylonian (Mesopotamian) arithmetic. Explain the base 60 notation that was used in Babylonia. Illustrate your explanation by showing how a couple of numbers were written, say 500 and $3/4$. Describe how addition, subtraction, multiplication, and division were performed (no illustration necessary).

Problem 2. [20] A classical proof. Prove that the sum of the three interior angles of any triangle is equal to two right angles. Your proof doesn't have to be perfectly formal, but point out where you use properties of parallel lines.

Problem 3. [10] We showed in class that the ratio of the diagonal to the side of a square is not equal to a ratio of two whole numbers, in other words, the diagonal of a square is not commensurable to the side of the square. Briefly explain why that result was important to the Pythagoreans. (Two or three sentences are sufficient.)

Problem 4. [20; 10 points each part] On Egyptian arithmetic.

- a. Illustrate how the Egyptian multiplication algorithm works by computing 45 times 97 (which is 4365).
- b. Illustrate how Egyptian division algorithm works by computing 4365 divided by 97 (which is 45).

Problem 5. [20; 5 points each part] True/false. For each sentence write the whole word “true” or the whole word “false”. If it’s not clear whether it should be considered true or false, you may explain in a sentence if you prefer.

- a. A common ancient approximation for the circumference of a circle was three times its diameter.
- b. The Euclidean algorithm to find the greatest common divisor of two numbers only works when the two numbers are both odd.
- c. An example of a Pythagorean triple of numbers is the triple 9, 40, 41.
- d. Whereas the Egyptians wrote on clay tablets, the Babylonians used papyrus.
- e. Euclid’s *Elements* includes constructions regular polygons with 3, 4, 5, 6, and 15 sides.