Math 105 History of Mathematics

First Test Answers

March 2013

Scale. 90–100 A, 80–89 B, 70–79 C. Median 95.

Problem 1. Essay. [30] Select *one* of the three topics A, B, and C.

Topic A. Contrast the level of Greek mathematics of the 4th century BC (the time of Eudoxus, Plato, Theaetetus, and ending with Euclid) with that of the Egyptians and Babylonians of the second millennium (the time of the Ahmes papyrus & Plimpton 322 tablet). Rather than trying to compare all of mathematics, choose only one main subject for comparison. Some example subjects: the Pythagorean theorem, formalism, numbers vs. geometry, algebra, similar figures, areas of circles.

Depending on which subject you choose, you'll have different things to say. For instance, the Egyptians apparently didn't know the Pythagorean theorem (at least we have no evidence that they did) while the Babylonians knew an used it, and the Greeks studied it intensively and proved it rigorously. On the other hand, Egyptians, Babylonians, and Greeks could all solve linear and quadratic algebraic problems, but you could mention how the Babylonians and Greeks could also deal with the quadratic ones by a geometric approach.

Topic B. We've discussed mathematics of Egypt, Babylonia, and Greece. Briefly summarize their transmission between cultures. Identify mathematics that may have been transmitted from one culture to one of the others. Explain why you think it may have been transmitted.

There seems to be no evidence of transmission between Egyptians and Babylonians; their number systems and computational methods were different and some of their knowledge of geometry was different. On the other hand, the Greeks learned much from both of the earlier cultures. For instance, Greeks used both unit fractions from the Egyptians and sexagesimal numbers from the Babylonians. There are several stories about early Greek mathematicians visiting both Egypt and Babylonia and learning mathematics there.

Topic C. On Babylonian (Mesopotamian) arithmetic. Explain the base 60 notation that was used in Babylonia. Illustrate your explanation by showing how a couple of numbers were written, say 500 and 3/4. Describe how addition, subtraction, multiplication, and division were performed (no illustration necessary).

Summary of points to make in your essay. Numerals were written with one symbol for 1 and another for 10. A place value system in base sixty was used so that six of the symbols for 10 was equal to one of the symbols for 1 in the next column to the left. Blanks were used for 0. For example, the number 500, which is 8 times 60 plus 20, in base 60 becomes 8,20 (actually eight 1-symbols followed by two 10-symbols).

Fractions were also written in base 60 so that 3/4 becomes 0;45 in base 60, but decimal points and zeros didnt appear, so 3/4 would look like 45. (The semicolons and commas we use to transcribe their numbers don't correspond to anything in their writing. We use them to help us understand what they had to tell by context.)

Since its a place-value system, addition, subtraction, and multiplication algorithms are the same ones we use in base 10, except done base 60, and so they're a bit more complicated. They didnt use long division, however. Instead they used tables to look up the reciprocal of the divisor and multiplied that by the dividend.

Problem 2. [20] A classical proof. Prove that the sum of the three interior angles of any triangle is equal to two right angles. Your proof doesn't have to be perfectly formal, but point out where you use properties of parallel lines.

This was an exercise in our fourth homework assignment. See the answers for it for Euclid's proof, but there are others.

A general proof should work for all triangles. A proof that begins with a right triangle is inadequate.

Problem 3. [10] We showed in class that the ratio of the diagonal to the side of a square is not equal to a ratio of two whole numbers, in other words, the diagonal of a square is not commensurable to the side of the square. Briefly explain why that result was important to the Pythagoreans. (Two or three sentences are sufficient.)

The important things to point out are that the Pythagoreans relied on whole numbers to explain everything, their motto was, after all, "all is number", but the ratio of the diagonal to the side of a square is not a ratio of numbers apparently contrary to that principle.

Problem 4. [20; 10 points each part] On Egyptian arithmetic.

a. Illustrate how the Egyptian multiplication algorithm works by computing 45 times 97 (which is 4365).

You either can start with a line with 1 and 45 in it and repeatedly double that line until you find a sum of powers of 2 in the left column that give 97, or you can start with a line with 1 and 97 and repeatedly double that line until you find a sum of powers of 2 giving 45. In either case, add the corresponding entries in the second column to find the product.

b. Illustrate how Egyptian division algorithm works by computing 4365 divided by 97 (which is 45).

Start with the line of 1 and 97 and repeatedly double it until you find a sum of numbers in the right column that add to 4365, then add the corresponding entries in the left column to find the quotient 45.

Problem 5. [20; 5 points each part] True/false.

a. A common ancient approximimation for the circumference of a circle was three times its diameter. *True.* 3 was the usual approximation for π.

b. The Euclidean algorithm to find the greatest common divisor of two numbers only works when the two numbers are both odd. *False.* It works for all numbers, both even and odd.

c. An example of a Pythagorian triple of numbers is the triple 9, 40, 41. *True.* $41^2 = 40^2 + 9^2$.

d. Whereas the Egyptians wrote on clay tablets, the Babylonians used papyrus. *False.* That's backward. Egyptians used papyrus and Babylonians used clay.

e. Euclid's *Elements* includes constructions regular polygons with 3, 4, 5, 6, and 15 sides. *True.* For instance, you did 15 yourself as a homework assignment.