Math 105 History of Mathematics

Aryabhata's trig table

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Aryabhata (b. 476) included a table of sines in his *Aryabhatiya* and a rule for constructing that table of sines. For Aryabhata, a sine was a half-chord in a circle of radius 3438 (the same radius Hipparchus had used centuries earlier). Thus, Aryabhata's sine for an angle θ equals $3438 \sin\theta$. His table is given in increments of $3^\circ\, 45'$ for angles strictly between 0° and 90°, but only increases in sines are given.

> *Stanza I, 10.* The twenty-four sine [differences] reckoned in minutes of arc are 225, 224, 222, 219, 215, 210, 205, 199, 191, 183, 174, 164, 154, 143, 131, 119, 106, 93, 79, 65, 51, 37, 22, 7.

If we denote these differences by $d_1, d_2, \ldots, d_{24}$, and their sums by $s_1 = d_1, s_2 = d_1 + d_2, \ldots, s_{24} = d_1 + d_2 + \cdots + d_{24}$, then the sums are the sines of various angles. The first entry, 225, gives $s_1 = 3438 \sin 3^\circ\, 45'$. Add to that the second, $d_2 = 224$, to give $s_2 = 449 = 3438 \sin 7^\circ\, 30'$. Add to that the third, $d_3 = 222$, to give $s_3 = 671 = 3438 \sin 11^\circ\, 15'$. And so forth.

Thus, if you've memorized the stanza, you can construct a table of sines for trigonometry since you can easily compute a sine from the previous sine and the sine difference: $s_n = s_{n-1} + d_n$.

In a later stanza, Aryabhata gives a rule for constructing the twenty-four sine differences. This stansa tells how to compute the differences d_n.

> *Stanza II, 12.* By what number the last sine [difference] is less than the first sine, and by the quotient obtained by dividing the sum of the preceding sine [differences] by the first sine, by the sum of these two quantities the following sine [differences] are less than the first sine.

As an equation, this rule says

$$(d_1 - d_{n-1}) + \frac{d_1 + d_2 + \cdots + d_{n_1}}{d_1} = d_1 - d_n$$

or, more simply,

$$d_n = d_n - s_{n-1}/225.$$

Below is an table of the values. It only depends on the two equations, $s_n = s_{n-1} + d_n$ and $d_n = d_n - s_{n-1}/225$, and the values in the first line. The numbers in the last column, d_n, are usually rounded down to the nearest integer, but sometimes rounded up to the next integer.

n	d_n	s_n	$s_n/225$
1	225	225	$225/225 = 1.0$
2	224	449	$449/225 = 2.0$
3	222	671	$671/225 = 3.0$
4	219	890	$890/225 = 4.0$
5	215	1105	$1105/225 = 4.9$
6	210	1315	$1315/225 = 5.8$
7	205	1520	$1520/225 = 6.8$
8	199	1719	$1719/225 = 7.6$
9	191	1910	$1910/225 = 8.5$
10	183	2093	$2093/225 = 9.3$
11	174	2267	$2267/225 = 10.1$
12	164	2431	$2431/225 = 10.8$
13	154	2585	$2585/225 = 11.5$
14	143	2728	$2728/225 = 12.1$
15	131	2859	$2859/225 = 12.7$
16	119	2978	$2978/225 = 13.2$
17	106	3084	$3084/225 = 13.7$
18	93	3177	$3177/225 = 14.1$
19	79	3256	$3256/225 = 14.5$
20	65	3321	$3321/225 = 14.8$
21	51	3372	$3372/225 = 14.9$
22	37	3409	$3409/225 = 15.2$
23	22	3431	$3431/225 = 15.2$
24	7	3438	