Assignment 1 answers
Math 105 History of Mathematics
Prof. D. Joyce, Spring 2013

1. Page 28, ex. 2: Use Egyptian techniques to multiply 34 by 18 and to divide 93 by 5.

1	18
′2	36
4	72
8	144
16	288
′32	576

Since $34 = 2+32$, therefore $34\cdot 18 = 36+576 = 612$.

1	5
2	10′
4	20
8	40
16	80′
$\overline{2}$	$2\,\overline{2}'$
$\overline{5}$	1
$\overline{10}$	$\overline{2}'$

After the first five lines, you've still have $93 - 80 - 10 = 3$ to go. You can halve the first line to get $2\frac{1}{2}$ of that 3 leaving $\frac{1}{2}$ to go. The last two lines take care of that. Since $93 = 80 + 10 + 2\frac{1}{2} + \frac{1}{2}$, the quotient is $16 + 2 + \frac{1}{2} + \frac{1}{10}$, that is, $18\,\overline{2}\,\overline{10}$.

2. Page 28, ex. 3: Use Egyptian techniques to multiply $\overline{2}\,\overline{14}$ by $1\,\overline{2}\,\overline{4}$.

There are two ways of doing this depending which of the two numbers you take to be the multiplier and which to be the multiplicand. Here's both.

′1	$\overline{2}\,\overline{14}$
′$\overline{2}$	$\overline{4}\,\overline{28}$
′$\overline{4}$	$\overline{8}\,\overline{56}$

1	$1\,\overline{2}\,\overline{4}$
′$\overline{2}$	$\overline{2}\,\overline{14}\,\overline{28}$
′$\overline{14}$	$\overline{14}\,\overline{28}\,\overline{56}$

In either case, add marked terms to get the sum $\overline{2}\,\overline{4}\,\overline{8}\,\overline{14}\,\overline{28}\,\overline{56}$.

Now, that's a fine answer, but it happens to simplify to 1. It's not clear how the Egyptians would have found the simplification, but they often used auxiliary computations. They might have put $\overline{14}\ \overline{28}\ \overline{56}$ in terms of seven $\overline{56}$'s as 'red auxiliaries' and simplified it to $\overline{8}$ as an intermediate computation.

3. Page 28, ex. 5: Show that the solution to the problem of dividing 7 loaves among 10 men is that each man gets $\overline{\overline{3}}\,\overline{30}$.

Start with the row 1 10 . You could take $\frac{1}{3}$ of that then twice that to get $\frac{2}{3}$ of it, but the Egyptians would have used a special $\frac{2}{3}$ table for that.

1	10
$\overline{\overline{3}}$	$6\,\overline{\overline{3}}'$
$\overline{10}$	1
$\overline{30}$	$\overline{3}'$

Now $6\,\overline{\overline{3}}$ is short of 7 by $\frac{1}{3}$, and the next two rows get you that remaining $\frac{1}{3}$. Since 7 is the sum of the right entries in the second and fourth rows, the left entries in those rows give the answer, namely, $\overline{\overline{3}}\,\overline{30}$.

4. Page 28, ex. 7. Multiply the Egypian fractions

1	$7\,\overline{2}\,\overline{4}\,\overline{8}$
2	$15\,\overline{2}\,\overline{4}$
′4	$32\,\overline{2}$
′8	63
′$\overline{\overline{3}}$	$4\,\overline{\overline{3}}\,\overline{3}\,\overline{6}\,\overline{12}$

Note that $\overline{\overline{3}}$ times 7 is $4\,\overline{\overline{3}}$. The product is the sum of the entries on the right of the marked lines, and that simplifies to $99\,\overline{2}\,\overline{4}$.

5. Page 28, ex. 8. Calculate 2 divided by 11 and 2 divided by 23 in the style of the Egyptians.

First, we'll show that 2 divided by 11 is $\overline{6}\,\overline{66}$.

$$
\begin{array}{rr}
1 & 11 \\
\overline{\overline{3}} & 7\,\overline{3} \\
\overline{3} & 3\overline{\overline{3}} \\
\overline{6} & 1\,\overline{2}\,\overline{3}' \\
\overline{11} & 1 \\
\overline{66} & \overline{6}' \\
\hline
 & 1\,\overline{2}\,\overline{3}\,\overline{6} \\
 & 2
\end{array}
$$

Next, we'll show that 2 divided by 23 is $\overline{12}\,\overline{276}$.

$$
\begin{array}{rr}
1 & 23 \\
\overline{\overline{3}} & 15\,\overline{3} \\
\overline{3} & 7\overline{\overline{3}} \\
\overline{6} & 3\,\overline{2}\,\overline{3} \\
\overline{12} & 1\,\overline{\overline{3}}\,\overline{4}' \\
\overline{23} & 1 \\
\overline{276} & \overline{12}' \\
\hline
 & 2
\end{array}
$$

6. Page 28, ex. 14: Solve problem 11 of the *Moscow Mathematical Papyrus*: The work of a man in logs; the amount of his work is 100 logs of 5 handbreadths diameter; but he has brought them in logs of 4 handbreadths diameter. How many logs of 4 handbreadths diameter are there?

The volume of logs of the same length are in a ratio of the squares of their diameters. Here, that means 16 of larger 5-handbreadth logs would equal 25 of the smaller 4-handbreadth logs.

We need to find how many of the smaller logs would equal 100 larger logs. So we need to solve the proportion

$$16 : 25 = 100 : \text{unknown}.$$

The Egyptians would have used the "rule of three" to solve the proportion. 100 times 25 divided by 16. That computation gives the answer $156\,\overline{4}$.

Math 105 Home Page at
`http://math.clarku.edu/~djoyce/ma105/`