1. [12] On limits of average rates of change. Let $f(x) = x^2 - 3x$.
   
   a. [4] Write down an expression that gives the average rate of change of this function over the interval between $x$ and $x + h$, and simplify the expression.
   
   b. [8] Compute the limit as $h \to 0$ of that average rate of change.
2. [10; 5 points each] On the intutitive concept of limit and continuity.

a. [5] Sketch the graph $y = f(x)$ of a function for which $\lim_{x \to 0} f(x)$ does not exist.

b. [5] Sketch the graph $y = f(x)$ of a function defined everywhere, the limit $\lim_{x \to 0} f(x)$ does exist, but $f$ is not continuous at $x = 0$. 
3. [10; 5 points each property] On asymptotes.
   a. Sketch the graph of a function $f$ such that
      \[
      \lim_{x \to 2^-} f(x) = \infty \quad \text{and} \quad \lim_{x \to 2^+} f(x) = -\infty.
      \]

   b. Sketch the graph of a function $f$ such that $\lim_{x \to \infty} f(x) = 1.$
4. [28; 7 points each part] Evaluate the following limits. If a limit diverges to $\pm \infty$ it is enough to say that it doesn’t exist.

a. \[ \lim_{x \to 1} \frac{x^2 - 1}{x^2 - 3x + 2} \]

b. \[ \lim_{x \to 1} \frac{x^2 - 4}{x^2 - 3x + 2} \]

c. \[ \lim_{x \to \infty} \frac{4x^3 - 2x}{9x^3 + 1} \]

d. \[ \lim_{x \to 0} \frac{4 \sin x}{5x} \]

Consider the limit \( \lim_{x \to 5} (2x - 3) \) which, of course, has the value 7. Since it has the value 7, that means that for each \( \epsilon > 0 \), there exists some \( \delta > 0 \), such that for all \( x \), if \( 0 < |x - 5| < \delta \), then \( |(2x - 3) - 7| < \epsilon \).

Let \( \epsilon = \frac{1}{2} \). Find a value of \( \delta \) that works for this \( \epsilon \). (Show your work.)

6. [10] Suppose that \( \theta \) is an angle between \( -\pi/2 \) and 0, and that \( \cos \theta = \frac{1}{2} \sqrt{2} \). Determine the value of \( \sin \theta \).
7. [15; 5 points each part] Suppose that \( \lim_{x \to \pi} f(x) = 5 \) and \( \lim_{x \to \pi} g(x) = 3 \). Evaluate each of the following limits, or explain why it doesn’t exist.

a. \( \lim_{x \to \pi} \frac{f(x)}{g(x)} \)

b. \( \lim_{x \to \pi} \frac{f(x)}{g(x) + 3 \cos x} \)

c. \( \lim_{x \to \pi} \sqrt{x + f(x)g(x)} \)