You may use a calculator. Leave your answers as expressions such as \( e^2 \sqrt{\frac{\sin^2(\pi/6)}{1 + \ln 10}} \) if you like. Show all your work for credit. Points for each problem are in square brackets.

1. [12] On limits of average rates of change. Let \( f(x) = 5x^2 + 4 \).
   
   a. [4] Write down an expression that gives the average rate of change of this function over the interval between \( x \) and \( x + h \), and simplify the expression.

   b. [8] Compute the limit as \( h \to 0 \) of that average rate of change.
2. [10; 5 points each] On the intuitive concept of limit and continuity.
   a. [5] Sketch the graph $y = f(x)$ of a function for which $\lim_{x \to 3} f(x)$ does not exist.

   

   b. [5] Sketch the graph $y = f(x)$ of a function defined everywhere, the limit $\lim_{x \to 3} f(x)$ does exist, but $f$ is not continuous at $x = 3$. 

   


3. [10; 5 points each property] On asymptotes.

a. Sketch the graph of a function $f$ such that

$$\lim_{x \to 2^-} f(x) = 0 \text{ and } \lim_{x \to 2^+} f(x) = -\infty.$$ 

b. Sketch the graph of a function $f$ such that $\lim_{x \to -\infty} f(x) = 1.$
4. [28; 7 points each part] Evaluate the following limits. If a limit diverges to $\pm \infty$ it is enough to say that it doesn’t exist.

a. $\lim_{x \to 1} \frac{x^2 - 4x + 4}{x^2 - x - 2}$

b. $\lim_{x \to 2} \frac{x^2 - 4x + 4}{x^2 - x - 2}$

c. $\lim_{x \to \infty} \frac{3x^2 - 2x + 1}{9x^3 + x}$

d. $\lim_{x \to 0} \frac{5x}{8 \sin x}$.

Consider the limit \( \lim_{x \to 3} (9 - 2x) \) which, of course, has the value 3. Since it has the value 3, that means that for each \( \epsilon > 0 \), there exists some \( \delta > 0 \), such that for all \( x \), if \( 0 < |x - 3| < \delta \), then \( |(9 - 2x) - 3| < \epsilon \).

Let \( \epsilon = \frac{1}{3} \). Find a value of \( \delta \) that works for this \( \epsilon \). (Show your work.)

6. [10] Suppose that \( \theta \) is an angle between \( \pi/2 \) and \( \pi \), and that \( \sin \theta = \frac{1}{2} \sqrt{2} \). Determine the value of \( \cos \theta \).
7. [15; 5 points each part] Suppose that \( \lim_{x \to \pi/2} f(x) = 4 \) and \( \lim_{x \to \pi/2} g(x) = 5 \). Evaluate each of the following limits, or explain why it doesn’t exist.

a. \( \lim_{x \to \pi/2} \frac{f(x) + g(x)}{f(x) - g(x)} \)

b. \( \lim_{x \to \pi/2} \frac{x}{g(x) - f(x) - \sin x} \)

c. \( \lim_{x \to \pi/2} \sqrt{(g(x))^2 + (f(x))^2} \)

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#1. [12]

#2. [10]

#3. [10]

#4. [28]

#5. [15]

#6. [10]

#7. [15]

Total