Name: ____________________________

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Math 130 Linear Algebra

Final Exam

December 2012

You may use a calculator and a sheet of notes. Leave your answers as expressions such as $e^2\sqrt{\dfrac{\sin^2(\pi/6)}{1+\ln 10}}$ if you like. Show all your work for credit. Points for each problem are in square brackets.

1. [12] Consider the three vectors $\mathbf{u} = (1,3,1)$, $\mathbf{v} = (4,2,-1)$, and $\mathbf{w} = (-3,1,2)$.

a. [8] Either prove that the $\mathbf{u}$ is in the span of the vectors $\mathbf{v}$ and $\mathbf{w}$, or prove that it is not. (There are several ways you can approach this question. Any one will do.)

b. [4] Are the three vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ linearly dependent, or linearly independent?

2. [16] Let A be the matrix $A = \begin{bmatrix} 2 & 2 & 2 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.

a. [4] Write down the characteristic polynomial $f(\lambda)$ for A.

b. [4] Determine the eigenvalues for A.

c. [8] For each of the eigenvalues of A, find the eigenspace of eigenvectors for that eigenvalue.

3. [15; 5 points each part] On dimension.

a. State the definition of the dimension of a vector space.

b. What theorem on vector spaces is required before this definition of dimension is a valid definition?

c. Let V be the vector space $V = \{(w, x, y, z) \in \mathbf{R}^4 \mid w = x + y + z\}$. Give an example of a 2-dimensional subspace W of V.

4. [15] If A is a 5×3 matrix, show that the rows of A are linearly dependent. (There are several ways that you can approach this. Be sure to write a clear and complete explanation.)

5. [15] Suppose that $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are vectors in $\mathbf{R}^3$. Prove that if $\mathbf{u}$ is orthogonal to both $\mathbf{v}$ and $\mathbf{w}$, then $\mathbf{u}$ is also orthogonal to $8\mathbf{v} + 13\mathbf{w}$.

6. [20; 4 points each part] Consider the three vectors in the plane $\mathbf{u} = (2, 1)$, $\mathbf{v} = (2, 3)$, and $\mathbf{w} = (6, 1)$.

a. Compute $||\mathbf{v} - \mathbf{w}||$.

b. Compute the inner product $\langle \mathbf{u} | \mathbf{v} \rangle$.

c. Let θ be the angle between $\mathbf{u}$ and $\mathbf{v}$. Find $\cos\theta$.

d. Which of the vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are parallel, if any?

e. Which of the vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are orthogonal, if any?

7. [10] Evaluate the following determinant using any method you like. Show all your work.

$$|A| = \begin{vmatrix} 1 & 0 & -3 & 0 \\ 0 & 1 & 0 & 1 \\ 5 & 3 & 0 & 0 \\ 0 & 0 & 2 & -1 \end{vmatrix}$$

#1.[12]	
#2.[16]	
#3.[15]	
#4.[15]	
#5.[15]	
#6.[20]	
#7.[10]	
Total	