Name: _____

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Math 130 Linear Algebra

First Test

October 2012

You may use a calculator. Leave your answers as expressions such as $e^2\sqrt{\dfrac{\sin^2(\pi/6)}{1+\ln 10}}$ if you like. Show all your work for credit. Points for each problem are in square brackets.

1. [10] Name two subspaces of $\mathbf{R}^2$ whose union is not a subspace of $\mathbf{R}^2$. (No need to give a proof.)

2. [12] Explain in a sentence or two why it is the case that if S is a linearly independent set of vectors in a vector space, then any subset S' of S is also a linearly independent set of vectors in that vector space.

3. [25] Consider the system of linear equations

$$\begin{array}{rcrcrcrcl} x & + & y & + & 3z & + & 2w & = & 7 \\ 2x & - & y & & & + & 4w & = & 8 \\ & & 3y & + & 6z & & & = & 6 \end{array}$$

a. [5] Write down the augmented matrix for this system.

b. [10] Use row operations to convert this matrix to echelon or reduced echelon form. (Your choice, but show your work.)

c. [10] Describe all the solutions for the original system of linear equations.

4. [25] Using only the axioms of vector spaces pertaining to addition prove the law of cancellation for addition:

$$\mathbf{u} + \mathbf{v} = \mathbf{u} + \mathbf{w} \text{ implies } \mathbf{v} = \mathbf{w}.$$

The axioms of vector spaces pertaining to addition are

1. Vector addition is commutative: $\mathbf{v} + \mathbf{w} = \mathbf{w} + \mathbf{v}$ for all vectors $\mathbf{v}$ and $\mathbf{w}$;
2. Vector addition is associative: $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$ for all vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$;
3. There is a vector, denoted $\mathbf{0}$ and called the *zero vector*, such that $\mathbf{v} + \mathbf{0} = \mathbf{v} = \mathbf{0} + \mathbf{v}$ for each vector $\mathbf{v}$;
4. For each vector $\mathbf{v}$, there is another vector denoted $(-\mathbf{v})$ such that $\mathbf{v} + (-\mathbf{v}) = \mathbf{0}$.

In your proof, do not use the operation of subtraction, but use the fourth axiom as it is stated.

Carefully write your proof so that you point out every use of every axiom. Fully parenthesize your expressions so that you can indicate when and where you use commutativity and associativity. Use full sentences and words like 'since' and 'therefore' to indicate the logical connections within your proof. You may want to sketch an outline or a draft of your proof on the back of one of the sheets of the test before writing up your final version.

5. [28; 4 points each] True/false. For each sentence write the whole word "true" or the whole word "false". If it's not clear whether it should be considered true or false, you may explain in a sentence if you like.

_____ **a.** The solutions to a system of linear equations form a vector space.

_____ **b.** Every subspace of a vector space must contain the zero vector, $\mathbf{0}$.

_____ **c.** The set of all polynomials with real coefficients forms a vector space over the field $\mathbf{R}$.

_____ **d.** Every symmetric matrix is an upper triangular matrix.

_____ **e.** If two nonzero vectors are linearly dependent, then one is a scalar multiple of the other.

_____ **f.** If the dimension of a vector space V is m, and the dimension of a vector space W is n, then the dimension of the product space $V \times W$ is mn.

_____ **g.** If there are n independent vectors in a vector space, then every spanning set for that vector space has at least n vectors.

#1.[10]	
#2.[12]	
#3.[25]	
#4.[25]	
#5.[28]	
Total	