Name: ____________________________

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Math 130 Linear Algebra

Second Test

November 2012

You may use a calculator and a sheet of notes. Leave your answers as expressions such as $e^2\sqrt{\dfrac{\sin^2(\pi/6)}{1+\ln 10}}$ if you like. Show all your work for credit. Points for each problem are in square brackets.

1. [16] Compute the inverse A^{-1} of the 3×3 matrix $A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$. You may use whatever method you like to find its inverse, but show your work. (If you have time at the end of the test, you may want to check that $AA^{-1} = I$ just to make sure you didn't make a mistake in your computations.)

2. [16; 8 points each part] Let A be the matrix $A = \begin{bmatrix} 1 & 5 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 2 & 14 \end{bmatrix}$ that describes a transformation $\mathbf{R}^5 \to \mathbf{R}^4$.

a. Determine the rank and the nullity of A.

b. Recall that the kernel, also called the null space, of a matrix A is the set of all solutions of the homogeneous system $A\mathbf{x} = \mathbf{0}$. Find a basis for the kernel A.

3. [28; 4 points each] True/false. For each sentence write the whole word "true" or the whole word "false". If it's not clear whether it should be considered true or false, you may explain in a sentence if you like.

___________ **a.** If $T : V \to W$ is a linear transformation, then T carries linearly independent subsets of V onto linearly independent subsets of W.

___________ **b.** The only square matrices A whose squares are the identity, $A^2 = I$, are the two matrices $A = I$ and $A = -I$

___________ **c.** The set of all linear transformations $V \to W$ between vector spaces over a field F forms another vector space over F.

___________ **d.** The vector space $M_{3,4}(\mathbf{R})$ of 3×4 matrices with entries in $\mathbf{R}$ is isomorphic to $P_{11}(\mathbf{R})$, the vector space of polynomials of degree 11 or less with coefficients in $\mathbf{R}$.

___________ **e.** If A is an invertible matrix, then $AB = AC$ implies that $B = C$.

___________ **f.** If the kernel of a transformation is $\mathbf{0}$, then the transformation is one-to-one.

___________ **g.** A linear transformation is invertible if and only if it is one-to-one and onto.

4. [16] Recall that two square matrices A and B are said to be *similar* or *conjugate* when there is an invertible matrix Q such that $B = Q^{-1}AQ$. We'll denote similar matrices $A \sim B$. Prove that this relation is transitive, that is, show that if one matrix is similar to another, and the second is similar to the third, then the first is similar to the third. Symbolically, that says $A \sim B$ and $B \sim C$ imply $A \sim C$.

5. [24; 4 points each] Identity or not identity. For each of the following equations involving scalars and matrices, write the word "identity" if the two sides of the equation are equal whenever both are defined, but write "not identity" if there exist scalars and matrices where the two sides of the equation are both defined, but they aren't equal. (You don't have to justify your answers.)

____________________ **a.** $A + (BC)D = B(CD) + A$.

____________________ **b.** $(A + B)(A - B) = A^2 - B^2$.

____________________ **c.** $(A^t + B)^t = A + B^t$. (The symbol t here indicates the transpose of a matrix.)

____________________ **d.** $(AB)^{-1} = A^{-1}B^{-1}$.

____________________ **e.** $(AB)^2 = A^2B^2$.

____________________ **f.** $(A^2 + I)(A^2 - I) = A^4 - I$.

#1.[16]	
#2.[16]	
#3.[28]	
#4.[16]	
#5.[24]	
Total	