Quiz
Math 130 Linear Algebra
D Joyce, October 2012

Scale. 9–10 A, 7–8 B, 5–6 C.

1. Given c a nonzero scaler, and given $\mathbf{v}$ and $\mathbf{w}$ two vectors. Prove that $c\mathbf{v} = c\mathbf{w}$ implies $\mathbf{v} = \mathbf{w}$.

The proof is fairly straightforward. Only axioms 6 are 5 are needed.

It is not necessary to move all the terms to the same side of the equation. If you do, then the proof triples in length, you'll need to use several more of the axioms, and you have to prove along the way that $-(c\mathbf{v}) = c(-\mathbf{v})$. Note that axiom 7 only applies to addition, not to subtraction.

Proof: Suppose that $c\mathbf{v} = c\mathbf{w}$. Since $c \neq 0$, and nonzero scalars have multiplicative inverses (that's a property of fields, not of vector spaces), therefore c^{-1} exist. Multiplying $c\mathbf{v} = c\mathbf{w}$ on both sides by c^{-1}, we find that

$$c^{-1}(c\mathbf{v}) = c^{-1}(c\mathbf{w}).$$

By axiom 6 for vectors spaces, that implies

$$(c^{-1}c)\mathbf{v}) = (c^{-1}c)\mathbf{w}).$$

But in a field, $c^{-1}c = 1$, so we can rewrite that equation as $1\mathbf{v} = 1\mathbf{w}$, and then by axiom 5, that simplifies to $\mathbf{v} = \mathbf{w}$. Q.E.D.

2. Given vector $\mathbf{v}$, prove $0\mathbf{v} = \mathbf{0}$.

There's more than one way to prove this, but all proofs will need to use axioms 2, 3, 4, and 8 somewhere.

Proof: In a field, $0 = 0+0$, therefore $0\mathbf{v} = (0+0)\mathbf{v}$. By axiom 8, that can be rewritten as

$$0\mathbf{v} = 0\mathbf{v} + 0\mathbf{v}.$$

Add the negation of $0\mathbf{v}$ to both sides of that equation. Note that $-(0\mathbf{v})$ exists by axiom 4.

$$-(0\mathbf{v}) + 0\mathbf{v} = -(0\mathbf{v}) + (0\mathbf{v} + 0\mathbf{v}).$$

Using axiom 2, we can rewrite that as

$$-(0\mathbf{v}) + 0\mathbf{v} = (-(0\mathbf{v}) + 0\mathbf{v}) + 0\mathbf{v}.$$

But $-(0\mathbf{v}) + 0\mathbf{v} = \mathbf{0}$ by axiom 4, so we have

$$\mathbf{0} = \mathbf{0} + 0\mathbf{v},$$

and finally, by axiom 3, we can rewrite that as $\mathbf{0} = 0\mathbf{v}$. Q.E.D.