Name: ___________________________

Mailbox number:___________________

Math 120 Calculus I

First Test

September 2013

You may use a calculator. Leave your answers as expressions such as $e^2\sqrt{\dfrac{\sin^2(\pi/6)}{1+\ln 10}}$ if you like. Show all your work for credit. Points for each problem are in square brackets.

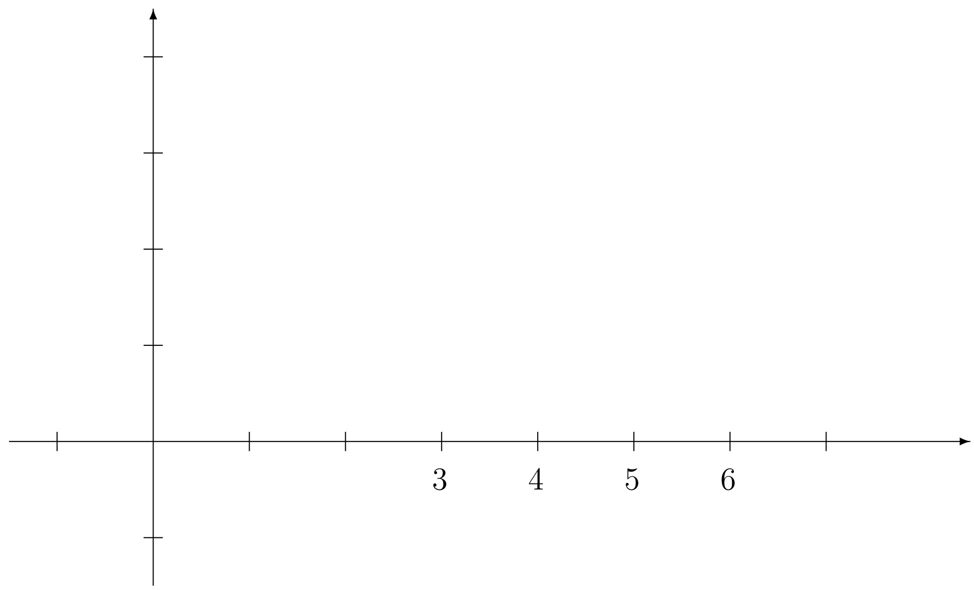
1. [12] On limits of average rates of change. Let $f(x) = x^2 - 3x$.

a. [4] Write down an expression that gives the average rate of change of this function over the interval between x and $x+h$, and simplifly the expression.

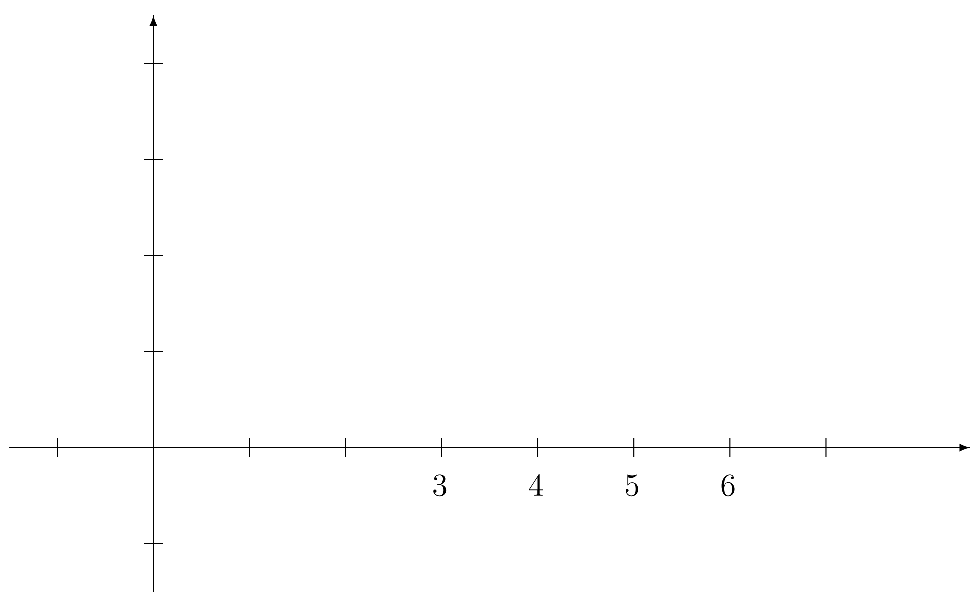
b. [8] Compute the limit as $h \to 0$ of that average rate of change.

2. [10; 5 points each] On the intutitive concept of limit and continuity.

a. [5] Sketch the graph $y = f(x)$ of a function for which $\lim_{x \to 0} f(x)$ does not exist.



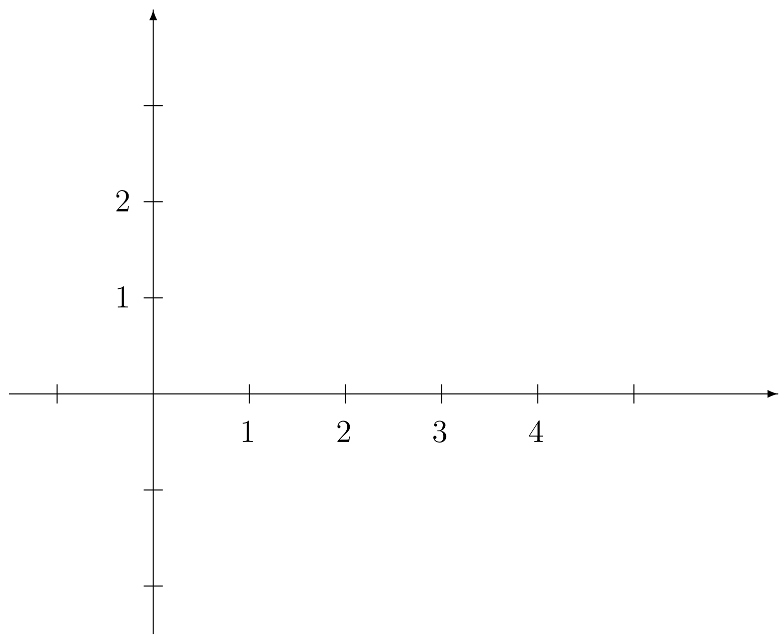
b. [5] Sketch the graph $y = f(x)$ of a function defined everywhere, the limit $\lim_{x \to 0} f(x)$ does exist, but f is not continuous at $x = 0$.



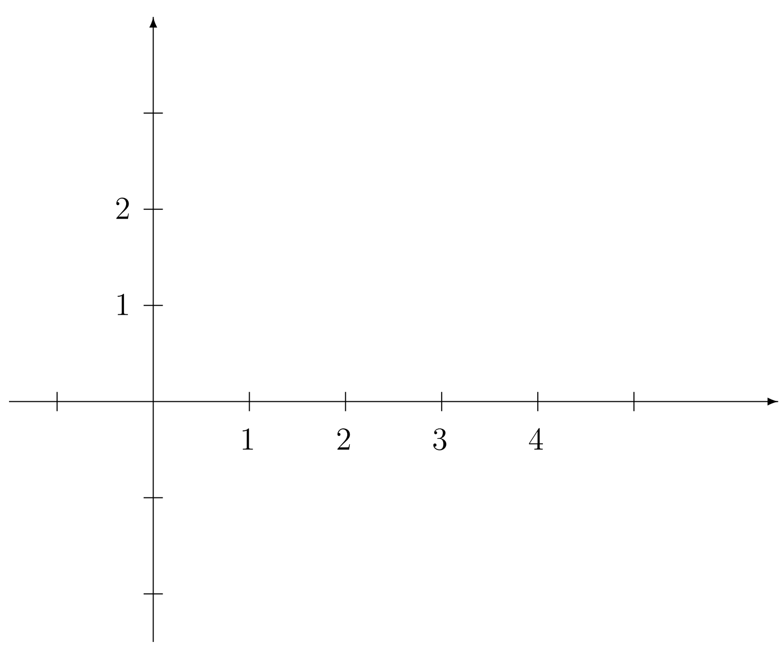
3. [10; 5 points each property] On asymptotes.

a. Sketch the graph of a function f such that

$$\lim_{x \to 2^-} f(x) = \infty \text{ and } \lim_{x \to 2^+} f(x) = -\infty.$$



b. Sketch the graph of a function f such that $\lim_{x \to \infty} f(x) = 1$.



4. [28; 7 points each part] Evaluate the following limits. If a limit diverges to $\pm\infty$ it is enough to say that it doesn't exist.

a. $\displaystyle\lim_{x\to 1} \frac{x^2-1}{x^2-3x+2}$

b. $\displaystyle\lim_{x\to 1} \frac{x^2-4}{x^2-3x+2}$

c. $\displaystyle\lim_{x\to \infty} \frac{4x^3-2x}{9x^3+1}$

d. $\displaystyle\lim_{x\to 0} \frac{4\sin x}{5x}.$

5. [15] On the formal definition of limit.

Consider the limit $\lim_{x \to 5}(2x - 3)$ which, of course, has the value 7. Since it has the value 7, that means that for each $\epsilon > 0$, there exists some $\delta > 0$, such that for all x, if $0 < |x - 5| < \delta$, then $|(2x - 3) - 7| < \epsilon$.

Let $\epsilon = \frac{1}{2}$. Find a value of δ that works for this ϵ. (Show your work.)

6. [10] Suppose that θ is an angle between $-\pi/2$ and 0, and that $\cos\theta = \frac{1}{2}\sqrt{2}$. Determine the value of $\sin\theta$.

7. [15; 5 points each part] Suppose that $\lim_{x\to\pi} f(x) = 5$ and $\lim_{x\to\pi} g(x) = 3$. Evaluate each of the following limits, or explain why it doesn't exist

a. $\lim_{x\to\pi} \frac{f(x)}{g(x)}$

b. $\lim_{x\to\pi} \frac{f(x)}{g(x) + 3\cos x}$

c. $\lim_{x\to\pi} \sqrt{x + f(x)g(x)}$

#1.[12]	
#2.[10]	
#3.[10]	
#4.[28]	
#5.[15]	
#6.[10]	
#7.[15]	
Total	