Outsourcing with Quality Competition: Insights from a Three Stage Game Theoretic Model

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April, 2008
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Abstract

Outsourcing decisions by organizations have strategic and operational implications. Strategically, understanding the market and competition is necessary to make effective outsourcing decisions. In this paper we recognize this concern and model the situation where an organization with quality and cost pressures and operational strategies may arrive at different outsourcing solutions based on competitor quality strategy traits. We develop a three-stage game-theoretic oligopolistic model based on differentiated product strategy and integrating quality expectations of the market. The model is solved for equilibrium points on price, outsourcing activity, and investments in quality. The results show that these decision factors are sensitive to market expectations and quality performance of competitors. Performance measures based on profitability and market share results are also presented within this model. Observations and insights are also presented.

Keywords: Outsourcing, Quality, Operations Strategy, Price, Game Theory
I. Introduction

Early in a product’s life cycle, with an emerging market, business organizations typically will perform most operational functions internally as vertically integrated entities. As the product life cycle matures, and competition increases, organizations will come under pressure to become more efficient. Efficiency goals may be met through aspects of cost cutting such as continuous improvement and business process reengineering efforts (Aitken, et al., 2003). Organizational efforts to help accomplish these tasks include strategic simplification of processes and focusing on organizational core competencies (Prahalad and Hamel, 1990; Quinn, 1999). A relatively straightforward method to achieve simplification and efficiencies of processes is to offload non-essential organizational activities, processes and functions to an outside party, i.e. outsourcing.

This process spin-off tendency is reinforced as features of the business resources, routines and activities – e.g. technologies and infrastructures in particular - become more commoditized and knowledge about business best practices diffuses as the product market expands and matures (Magnani, 2006). Under this development, organizations are almost coerced to become more concentrated on selling their core competencies which demonstrate a definite comparative and competitive advantage in the product market.

Even though outsourcing has become a concept with significant definitional dissonance, it typically is concerned with an issue of product quality vs. costs (Aron, et al., 2008; Reyniers and Tapiero, 1995). In this paper, we investigate this classical dilemma in a game theoretic perspective. By setting up a model developed in the product differentiation literature in economics, we analyze a case where a quality competing oligopolistic firm outsourcing its inputs from competitive subcontractors decides on key variables spanning over three stages; product quality (via R & D investment), the rate of outsourcing, and finally the price of the product.

This problem is modeled and analyzed using a game-theoretic concept defined as Subgame Perfect Equilibrium through backward induction. This approach to the issue of outsourcing has several advantages over other models for understanding the dilemma of
quality vs. costs tradeoffs. First, this model explicitly introduces the realities in which there are quality differences of a product, manufacturers with brand names compete in terms of qualities as well as in terms of prices. Second, we find an outsourcing strategy is generally adopted by a firm that is less sensitive to product quality. That is, its operations strategy would put greater emphasis on cost reduction and metrics rather than a quality based production and operations strategy. Hence this firm should be aware of the fact that a possible degradation of a quality through outsourcing might affect its market share through quality competition with other oligopolistic firms. This feature of the consequences of outsourcing may be captured and analyzed by our model. Third, outsourcing may affect a customer’s preference and utility. The responses of customers to outsourcing are incorporated using economically modeled customer utility functions. Fourth, outsourcers faced with quality performance competition are generally under an oligopolistic market structure, while subcontractors with easy entry and exit who depend on lower wages are under strong competitive pressures can be evaluated.

The remainder of this paper begins with some background discussion that motivates the problem and issues faced in outsourcing. Then the three stage game theoretic model is introduced. We then provide some characteristics of the solution to this model. Some initial analysis and insights through an experimental parametric analysis provides some initial insights follow. Finally a conclusion with summaries of our findings and extensions for future investigation are provided.

II. Outsourcing and the Quality-Cost Competitive Environment

Despite popular usage of the term ‘outsourcing’ by media and the public, there still remains some confusion over the definition of outsourcing. According to Bhagwati et al (2004), during the 1980s, ‘outsourcing’ typically meant a situation when business firms expanded their purchases of manufactured (physical) inputs from outside rather than in-house. For instance, car companies may purchase their essential physical inputs such as tires, brake system, window cranks, passenger seats, and so on from another company which may have a long term contract with the original firms. Currently, however, in which commodity (whether it is a good or service) information is easily accessible around the globe with a click on a mouse, the meaning of ‘outsourcing’ expands into a specific segment of the growing international trade in services (Aron et al., 2008). This
environment alters the definition slightly to take a form of purchase of services abroad, either through transactions by firms or with direct consumption purchases by individuals\textsuperscript{1}.

Delegating a specific project or a responsibility to an outside third-party entity, sub-contractor, a specialized firm, or even an overseas production development unit, whether it is a product or service, is what we will define as outsourcing. It generally occurs for production process activities. The original producer would employ an outside producer and contracts to make portions or components of a product (or some process of production) using its own resources. There is ample evidence that partial outsourcing is a significant market practice where it was recently estimated that only the information technology (IT) outsourcing practices to offshore locations represented a $178 Billion market in 2005 (Chakrabarty et al., 2006). In the mobile telephone industry, Nokia, Motorola, and Ericsson outsource mobile handsets at between 15%-40% rates (Economist, 2002). Part of the argument for outsourcing is the decrease in production costs. Some estimates have shown that effective outsourcing can reduce costs by overall production costs by 20-40% per year (Domberger, 1995; Domberger et al 2002).

While cost saving may be a dominant reason behind the rush toward outsourcing for many firms, potential quality degradation resulting from unscrutinized offshore subcontracting may become a highly detrimental threat as in the case of Mattel in the toy industry\textsuperscript{2}. The recent case of mass recalls of Mattel toys provide an excellent example of the dilemma faced by an outsourcing firm. The toys were made by Chinese local firms through a typical offshore outsourcing arrangement, called Original Equipment Manufacturing (OEM). In general, American toy makers design and process a product and then transfer a knowledge regarding the product to subcontractors in China, buy back the products, and sell them to domestic customers with its own brand. Outsourcing via

\textsuperscript{1}The WTO categorizes four different ways in which services can be traded. Mode 1 refers to arm’s-length supply of services with both of the trading partners remaining in their respective locations. This type of trade has come into existence through IT revolution and unlike goods trade, it could not readily be subjected to customs inspection. In Mode 2, as in tourism and medical care, service recipients move to the location of service providers. Mode 3 categorizes a case where the service provider establishes a commercial presence in another country as in banking and insurance. In general, this type of trade requires foreign direct investment. In Mode 4, as in construction and consulting, service providers move to the location of the service buyer. When they discuss the problem of “outsourcing”, most economists and policy makers have meant trade in Mode 1 services. (Bhagwati et al(2004))

offshore production enabled the domestic firms to enjoy low production costs with much less managerial underpinnings. U.S. producers did not fully recognize, however, that a lower cost ultimately led to a quality degradation, to such an extent that the final product couldn’t keep up with the domestic safety standard for the product, which happened to grow higher with income; local producers in China could not see any incentives to follow high quality standards set in the US, since the Chinese government was much less stringent to quality standards and the original producer in the US did not elaborate to monitor the performance. The worldwide recall of nearly 20 million lead-painted toys further damaged outsourcing from China, coming after pet food found to contain melamine, toothpaste tainted with diethylene glycol and tires that separate at the treads. This is a case where a low quality (cost-driven) producer underestimated the value of monitoring costs, inducing more equilibrium rate of outsourcing, hence naively enjoyed an expansion of market and high profit.

Part of the outsourcing quality degradation arguments and difficulties not only relate to product quality, but other dimensions of product quality such as effective customer service and product delivery (Tan et al., 1998). This cost/quality tradeoff for outsourcing is a common enough occurrence and concern that particular attention to this strategic decision and tradeoff is warranted (Reyniers and Tapiero, 1995; Wadhwa and Ravindran, 2007).

In economics, outsourcing practices of firms are characterized as an important topic related to the boundaries of the firm within the framework of the theory of organization. Hence there exist many interesting and important papers using the theory of transaction costs, contract theory, industrial general equilibrium, vertical and horizontal integration and differentiation, among others to address this issue and supply chain relationships. The research and theoretical frameworks we focus on in this paper has seen relatively little study. There are a few oligopolistic models using Cournot and Bertrand competition which do relate to our research. Kamien, Li, and Samet (1989) applies an auction approach in a duopoly framework with Bertrand (price) competition, allowing subcontracting to each other. Shy and Stenbacka (2004) explores the strategic nature of outsourcing under Cournot competition in the final goods market. Even though these and other papers have investigated outsourcing from a game theoretic perspective,
the inclusion of various customer expectations and quality dimensions, have not been completed. Outsourcing in a three stage game framework has not yet seen any development in the research literature. We take advance this area of study by providing a three-stage game model that provides significant and robust managerial insight.

III. The model

To begin the model development, we consider a duopoly model with vertical differentiated products introduced by Mussa and Rosen (1978). It is assumed that customers are heterogeneous in their marginal valuation of product quality, or quality expectations which is denoted by $v$. There is a population of customers whose total number is equal to $N$ and each customer buys at most one unit. For simplicity, we assume that a uniform distribution of customer types and their quality expectations is given by $v_i \sim U[0, N]$. For a given price $p_j$ and level of product quality $\theta_j$ with $j = \{A, B\}$, $A$ and $B$ represent two competing firms where firm $B$ is seeking to make an outsourcing decision, the net utility for a customer with quality expectation $v_i$ is

$$u(v_i) \overset{\text{def}}{=} \theta_j v_i + \theta - p_j$$ (1)

In this paper the quality of each firm is determined through its product innovation which is investment in the quality of its product. We assume that firm $j$ chooses its quality $\theta_j$ at period one with a research and development (R&D) cost of $\Theta(\theta_j) \overset{\text{def}}{=} \beta(\theta_j - \theta)^2$ if $\theta_j > \theta$, 0 otherwise. The R&D process or quality improvement investments are not always required since each firm is assumed to be endowed with a minimum quality level $\theta $, which is assumed to be large enough to cover the market.

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3 A unit may be representative of a single product, a lot of products, or even a long term contract. The expectation here is that for this ‘unit’ the quality requirements and expectations are the same.

4 Research and Development costs would be used to improve product quality, this cost may also incur costs for improving process quality through Six-Sigma, Total Quality Management, or a ISO 9000-like programs.

5 Here $\theta$ is assumed to draw constant utility for customers independent of their quality expectations.

6 A covered market means that the minimal expectations of all customers are met with this level of quality. Some papers adopt a vertical differentiated product model with covered market. See Crampes and...
Producing final products requires various inputs which can be obtained by different production modes. We assume that each final product of firm $j$ requires $I$ inputs which the total number of inputs is normalized to one (competing products are assumed to have equal levels of inputs). For acquiring each input, two options are open to firm $j$: in-house production and outsourcing via outside subcontractors. $r_x \in [0,1]$ denotes the percentage of inputs produced by the outside subcontractors. We assume that outside subcontractors are specialized in producing inputs for the products in a competitive market with normalized zero unit costs. Therefore, the price of outsourced inputs is assumed to be zero. On the other hand, the unit cost of in-house production of an input is $c$ for firm $j$ ($j=A, B$). This assumption reflects the assumption that in-house production is less efficient, more costly, than outsider subcontractors and one of the main factors for firms to outsource their inputs production to lower its unit cost of production. However, outsourcing also incurs two different types of costs, expected quality performance costs and monitoring costs.

To comment briefly on the two types of costs, we first assume that the product produced partially by outside subcontractors entails reduction in expected (perceived) quality performance, denoted by $\alpha(r_x) = \alpha r_x$, which is assumed to be constant across all customers. The parameter $\alpha$ captures the marginal impact on the expected quality performance. We assume that an increase in the outsourcing rate has a negative impact on expected quality performance due to increased difficulty in managerial coordination and oversight by the producer or the incomplete nature of contracts on the quality of inputs production between the outsourcer and the outside subcontractors. We assume that these costs are the same across all final customers or at least independently distributed with the valuation of customers for the product. The second type of cost incurred with outsourcing is a firm’s increased monitoring cost denoted by $m(r_x) = \gamma r_x^2$ to ensure the quality of inputs produced by outside subcontractors. It is assumed to be an increasing convex function of the rate of outsourcing. The parameter $\gamma$ captures the marginal impact on the outsourcing monitoring costs.

In order to emphasize explicitly the strategic effect of outsourcing on competition and quality level we carry out this task within a framework where only one firm with low quality product (i.e., firm B) is able to outsource its production. Given the outsourcing rate \( r_B \) the customer’s net utility becomes

\[
u(N) = \begin{cases} 
\theta_A v_i + \theta - p_A & \text{if the customer buys product A} \\
(\theta_B v_i + \theta - \alpha r_B) - p_B & \text{if the customer buys product B with the outsourcing rate } r_B 
\end{cases}
\]

(2).

To help solve the outsourcing problem we consider a three-stage game with the following stages:

**Stage one**: Each original production firm (A and B) chooses its R&D or quality investment to improve its level of quality \( \theta_i \).

**Stage two**: Firm B decides the rate of outsourcing the production of its inputs \( r_B \), thereby firm B produces \( 1 - r_B \) and buys \( r_B \) portion of inputs from the outside subcontractors.

**Stage three**: Each firm chooses its price \( p_j \) to maximize its profit.

**IV. Solving for Equilibrium in the Three-Stage Game**

To find the Subgame Perfect Nash equilibrium (SPNE), we begin with period three.

**1. Stage Three: price competition with given quality and outsourcing rate**

In Stage Three, with given quality levels \( \theta_A, \theta_B \) and the outsourcing rate \( r_B \) the two firms (A and B) compete for customers in terms of pricing. Even though there are pricing expectations, it is assumed that due to competitive reasons the quality expectations are at a minimal level or higher in this quality competitive environment. When customers make their purchase decision, they choose the option that yields the highest net utility. We consider the case of \( \theta_A > \theta_B \), without loss of generality, where firm A is competing primarily on maintaining higher quality than its competitors and firm B will have the option to further improve its quality or further compete on pricing. In this
case, a customer’s optimal choice between products or companies for a given price and quality level can be divided as follows:

\[
\frac{p_A - p_B - \alpha r_B}{\theta_A - \theta_B} \leq v \quad \text{purchase product from Firm A}
\]

\[
v < \frac{p_A - p_B - \alpha r_B}{\theta_A - \theta_B} \quad \text{purchase product from Firm B}
\]

Thus, for a given price and quality level, there will be a customer with quality expectations denoted by \( \hat{v} \) that is indifferent between buying products from firm \( A \) and \( B \) if \( \theta_A \hat{v} + \theta_B = (\theta_B v + \theta_A - \alpha r_B) - p_B \). This feature of the self-selection choice of customers between products is shown in Figure 1.

![Figure 1 about here](image)

Each firm’s demand then can be described as

\[
q_A = N - \hat{v} = N - \frac{p_A - p_B - \alpha r_B}{\theta_A - \theta_B} \quad \text{and} \quad q_B = \hat{v} = \frac{p_A - p_B - \alpha r_B}{\theta_A - \theta_B}
\]  

(3).

Given the marginal cost \( c \), firm \( A \), which is assumed will not outsource, maximizes its profit by

\[
\max_{p_A} \left( p_A - c \right) \left( N - \frac{p_A - p_B - \alpha r_B}{\theta_A - \theta_B} \right) - \beta (\theta_A - \theta)^2
\]

(4a).

Similarly, given the reduced marginal cost via outsourcing, firm \( B \) maximizes its profit by

\[
\max_{p_B} \left( p_B - (1 - r_B) c \right) \left( \frac{p_A - p_B - \alpha r_B}{\theta_A - \theta_B} \right) - \beta (\theta_B - \theta)^2 - \gamma r_B^2
\]

(4b).
By solving each team’s profit maximization problem, we derive the equilibrium
price \((p^*)\) and market share \((q^*)\) as follows.

\[
p_A^*(\theta_A, \theta_B, r_B) = \frac{1}{3} \left[ (\alpha r_B + (3-r_B)c + 2N(\theta_A - \theta_B)) \right] \tag{5a}
\]

\[
p_B^*(\theta_A, \theta_B, r_B) = \frac{1}{3} \left[ (3c - r_B(\alpha + 2c) + N(\theta_A - \theta_B)) \right] \tag{5b}
\]

\[
q_A^*(\theta_A, \theta_B, r_B) = \frac{1}{3} \left[ 2N + \frac{r_B(\alpha - c)}{\theta_A - \theta_B} \right] \tag{6a}
\]

and

\[
q_B^*(\theta_A, \theta_B, r_B) = \frac{1}{3} \left[ N - \frac{r_B(\alpha - c)}{\theta_A - \theta_B} \right] \tag{6b}
\]

The corresponding equilibrium profits are as follows:

\[
\pi_A^*(\theta_A, \theta_B, r_B) = \frac{r_B(\alpha - c) + 2N(\theta_A - \theta_B)}{9(\theta_A - \theta_B)} - \beta(\theta_A - \theta_B)^2 \tag{7a}
\]

and

\[
\pi_B^*(\theta_A, \theta_B, r_B) = \frac{r_B(\alpha - c) - N(\theta_A - \theta_B)}{9(\theta_A - \theta_B)} - \beta(\theta_B - \theta_B)^2 - \gamma r_B^2 \tag{7b}
\]

2. **Stage Two: Choice of production mode. How much to outsource?**

We now turn to firm B’s optimal choice on its rate of outsourcing. The objective
is to simply maximize equation 7(b) with respect to \(r_B\), which yields the following
necessary first-order condition:

\[
\frac{\partial \pi_B^*(\theta_A, \theta_B, r_B)}{\partial r_B} = \frac{2 \left[ r_B(\alpha - c)^2 - (9\gamma r_B + (\alpha - c)N)(\theta_A - \theta_B) \right]}{9(\theta_A - \theta_B)} = 0 \tag{8}
\]

which gives us
\[ r^*_b(\theta_A, \theta_B) = \frac{(\alpha - c)N(\theta_A - \theta_B)}{(\alpha - c)^2 - 9\gamma(\theta_A - \theta_B)} \] (9).

In other words, firm B produces a \(1 - r^*_b\) proportion of its product inputs in-house and buys a \(r^*_b\) proportion of its product inputs from outsourced subcontractors. To ensure concavity of the profit function with respect to \(r_b\), we require the following second order condition:

\[ \frac{\partial^2 \pi^*_b(\theta_A, \theta_B, r_B)}{\partial r_B^2} = -2\gamma + \frac{2(\alpha - c)^2}{9(\theta_A - \theta_B)} < 0 \] (10)

From this second order condition we derive

\[ c - 3\sqrt{\gamma(\theta_A - \theta_B)} < a < c + 3\sqrt{\gamma(\theta_A - \theta_B)} \] (11)

which means the marginal disutility of outsourcing of customers on the expected quality performance should not be too large or too small when compared with the magnitude of the firm’s unit production cost, to have an interior solution of the optimal rate of outsourcing. Also, incorporating the boundary condition of the outsourcing rate \((0 \leq r^*_b \leq 1)\), we have

\[ \frac{1}{2} \left( 2c + N(\theta_A - \theta_B) - \sqrt{\theta_A - \theta_B} \sqrt{36\gamma + N^2(\theta_A - \theta_B)} \right) \leq \alpha \leq c \] (12).

Combining these two conditions from eq. (11) and (12) together, we derive the following constraint for the solution.

\[ \max \left\{ c - 3\sqrt{\gamma(\theta_A - \theta_B)}, \frac{1}{2} \left( 2c + N(\theta_A - \theta_B) - \sqrt{\theta_A - \theta_B} \sqrt{36\gamma + N^2(\theta_A - \theta_B)} \right) \right\} \leq \alpha \leq c \] (13).

3. Stage one: choice of quality investment and quality level
At period one, the firms choose the level of quality to maximize their own profits, which are given respectively by

\[
\pi^*_A(\theta_A, \theta_B) = \frac{N^2(\theta_A - \theta_B)\left((\alpha - c)^2 - 6\gamma(\theta_A - \theta_B)\right)^2}{\left((\alpha - c)^2 - 9\gamma(\theta_A - \theta_B)\right)^2} - \beta(\theta_A - \theta)^2 \tag{14a}
\]

and

\[
\pi^*_B(\theta_A, \theta_B) = \frac{9N^2\gamma^2(\theta_A - \theta_B)^3}{\left((\alpha - c)^2 - 9\gamma(\theta_A - \theta_B)\right)^2} - \beta(\theta_B - \theta)^2 - \gamma \left(\frac{\alpha - c)N(\theta_A - \theta_B)}{(\alpha - c)^2 - 9\gamma(\theta_A - \theta_B)}\right)^2 \tag{14b}
\]

The best response function can be derived in terms of its own first order conditions.

\[
\frac{d\pi^*_A(\theta_A, \theta_B)}{d\theta_A} = \frac{1}{9} \left(4N^2 - 18\beta(\theta_A - \theta) + \frac{2(\alpha - c)^6 N^2}{\left((\alpha - c)^2 - 9\gamma(\theta_A - \theta_B)\right)^2} + \frac{3(\alpha - c)^4 N^2}{\left((\alpha - c)^2 - 9\gamma(\theta_A - \theta_B)\right)^2}\right) \tag{12a}
\]

\[
\frac{d\pi^*_B(\theta_A, \theta_B)}{d\theta_B} = \frac{1}{9} \left(-N^2 - 18\beta(\theta_B - \theta) + \frac{(\alpha - c)^4 N^2}{\left((\alpha - c)^2 - 9\gamma(\theta_A - \theta_B)\right)^2}\right) \tag{12b}
\]

V. Empirical Investigation and Discussion

Due to the complex algebraic expressions in equations (12a) and (12b), it is mathematically intractable to derive the optimal equilibrium quality choice of the firms in explicit functional forms. To understand the decision mechanisms of the firms and the results in from the game equilibrium numerically, we run some analyses. In this empirical analysis, we compare the effects on the decision variables (outsourcing percentage \(r^*_g\), price \(p^*_A, p^*_B\), and investment in quality programs \(\theta^*_A, \theta^*_B\)) by the exogenous parameters, \(\alpha\) (factor of marginal decreases in quality due to outsourcing), \(c\) (unit product cost), and \(\gamma\) (factor of marginal increase in monitoring costs due to
Tables 1, 2 and 3 summarize these results, where we fixed other parameter values at $N = 1$, $\beta = 1$, and $\theta = 1$, respectively.

1. **The effect of the marginal disutility of customers on outsourcing (expected quality performance)**

The results in Table 1a and Table 1b show that as the customers become more quality sensitive about outsourcing, their perceptions of outsourcer quality, the optimal rate of outsourcing, equilibrium quality and price difference between the two firms will be lowered. In this situation, the relative market share and profit of the high quality producer (firm A) will tend to increase as expected since quality competition tends to increase as this customer quality sensitivity increases. This result can be easily explained through intuition. As the customers become more sensitive about the quality degradation from outsourcing, low quality, low price producer seeking through outsourcing will be discouraged. Thus, there will be pressures or incentives to increase quality investments and raise price. Hence the equilibrium price difference between the quality oriented (A) firm and the price/cost oriented (B) firm will be lessened. If the low quality producer be oriented toward quality competition, the high quality producer will gain, hence its optimal market shares ($q^*$) and profits ($p^*$) will be relatively higher as the market’s (customer’s) quality expectations increase. Therefore, from this reasoning, we can make the following observation:

**Observation 1** The more the customers are sensitive to outsourcing quality, quality and price competition becomes more intense, and the high quality producer will gain.

We also see that customers can expect the prices of their products to increase as their sensitivities increase, since the optimal prices for maximizing the profits for both firms tend to increase. Even though the prices increase for both firms, firm B, the cost-driven firm, will lose market share as its prices get closer to the quality leading firm A, as shown by the ratio ($q^*_A/q^*_B$) in Table 1b. Overall, the optimal profitability of firm A will tend to increase in this environment. A strategic marketing outcome of this observation is that
firm A should try to further differentiate itself on quality and reinforce the perception in customers that firms that outsource will have poorer quality performance.

2. The effects of unit production cost difference from subcontractors

We have assumed that subcontractors are under competitive pressure and normalized their unit production cost to be zero (guaranteeing a lower production cost than the original firms A and B). In this situation, increases in the unit production cost of by quality competing firms (firm A) presents the case where outsourcing accrues greater benefit in production cost. Tables 2a and 2b shows the effects on firms’ equilibrium decisions of this manipulation.

These second stage results show a slightly different result from the previous stage. As the unit production cost \( c \) increases, the potential benefit of outsourcing tends to also increase. Hence, the optimal outsourcing rate \( r_B^* \) of the low quality, cost-driven, producer firm (B) will tend to increase. The equilibrium quality differential (represented by the ratio \( \left( \frac{\theta_A^* + \theta}{\theta_B^* + \theta} \right) \) in Table 2b), will be higher since there will be more outsourced products. In this case, the outsourcing firm B, the cost-competitive, lower quality producer, will expand their market share \( q_B^* \) in Table 2a) and will have a higher relative equilibrium profit \( \pi_A^*/\pi_B^* \) in Table 2b). But the effects on the equilibrium prices \( p^* \) are less clear. The direct effect of an increase in the unit production cost will be an increase in product prices. The indirect effect via outsourcing, however, will induce a price decrease. The relative effects of these two contradicting forces in equilibrium depend on the intensity of quality competition. The table shows that if the difference of the unit production cost is relatively minor between the original product firm (B) and the subcontracting firms, and hence outsourcing savings, are relatively insignificant, increases in the unit production cost will bring lower equilibrium differences in prices. This result occurs because outsourcing contributes little to quality in this stage, the lower quality producer will be more sensitive to price caused by a cost increase as in-house production is much more significant. As the difference of the unit production cost
becomes greater between the outsourcer and subcontracting firms, the lower quality producer (firm B) will become less sensitive to an increase in the cost of in-house production. This situation will ultimately result in an increase in the equilibrium price difference. This reasoning leads to an implicit proposition that there might be a level of the cost incentive of outsourcing resulting in a minimum difference in equilibrium prices in oligopolies under quality competition. Also, we can observe that as production costs increase, the quality oriented firm (A) will tend to invest more in its quality initiatives ($\theta_A$ in Table 2) on an absolute level and when compared to firm ($\frac{(\theta_A^* + \theta)}{(\theta_B^* + \theta)}$ in Table 2b).

**Observation 2** As the cost benefit of outsourcing becomes larger ($c$ becomes larger), the equilibrium quality difference will be greater, and the outsourcing low quality producer will gain.

**Remark 2-1** In the initial stage of outsourcing, where in-house production is dominant, cost increase in the outsourcer’s industry will be associated with lowered price differences. But in the stage of significant percentage of outsourcing, cost increase will bring clear quality difference and hence price difference.

**Remark 2-2** Whether an increase in outsourcing, resulted from a cost increase in the outsourcer’s industry, will bring an increase in price difference between the high quality producer and the lower quality producer, is an empirical and open question.

We can postulate that it price differences may increase or decrease in difference (the $p_A^*/p_B^*$ ratio in Table 2b) amongst the firms may be estimated by empirically finding the minimum level of price difference over the span of various degrees of outsourcing. Thus, if firm B feels that the pricing difference can be improved if they are not at this minimal difference level by manipulating the outsourcing rate ($r_B^*$). If the current stage of outsourcing is believed to be below the rate of that minimum level, an increase in the rate
of outsourcing due to the cost benefit will bring weakened price competition. Otherwise, the price gap will be deeper.

3. **The effects of monitoring costs**

It is intuitively clear that as the outsourcing monitoring costs increase (i.e., as $\gamma$ becomes larger), the equilibrium rate of outsourcing ($r^*_b$) will decrease. The results shown in Tables 3a and 3b of the first stage of the three stage game, verify this intuition. These table results show that as quality monitoring costs for outsourcing increase, the optimal rate of outsourcing decrease, leading to a decrease in quality and price difference. In this scenario, the relative market share of the quality oriented producer (A) increases ($q^*_A$ in Table 3a) and hence will ultimately gain ground on B. This situation occurs as the equilibrium investment in quality by A ($\theta^*_A$ and $(\theta^*_A + \theta)/(\theta^*_B + \theta)$) actually decrease. Another surprising finding here is that even if monitoring costs increase, the equilibrium profitability increases slightly. This increase in profitability seems to occur from increases in equilibrium prices for A ($p^*_A$) and B ($p^*_B$).

**Observation 3.** *Higher quality outsourcing monitoring costs will lower the optimal rate of outsourcing, quality and price differences, and will be beneficial to the quality-oriented producer.*

One interesting, but very intuitive operations strategy implication from this result is that all other things being equal, industries with a lower level of standardization (higher customization and hence in need of more careful observation) will be less prone to outsourcing and a high quality producer will ultimately gain.

VI. **Conclusion.**

In this paper, we analyzed firm decision problems of outsourcing in light of operations strategy and product differentiation by setting up a three stage game relating to factors of quality competition, outsourcing, and price competition, respectively. By incorporating the effects of outsourcing on customers’ welfare, quality improvement (R&D) costs, and quality outsourcing monitoring costs, we were able to derive and observe some interesting results relating to decisions on outsourcing. We clearly observe
that there is a conflict and tradeoff associated with production cost benefits and quality degradation typically associated with outsourcing in a quality competitive market. The introduction of quality into an outsourcing decision framework is necessary in quality competitive environment, but the results have implications in most decision environments, even in situations where price and quality are less tradeoff oriented.

By borrowing the basic insights and theoretical developments from vertical differentiation literature within economics, and by incorporating an outsourcing decision problem of the firms into a classical quality and price competition model, we were able to make a number of insights. First, the outsourcing decision will amplify the difference of firms in the relative investments in quality programs. More precisely, based on our three-stage game theoretic model, firms will choose more divergent quality management strategies when faced with outsourcing possibilities. Second, if customers and the market becomes more sensitive to (perceptions of poor quality increase) with outsourced products, quality and price competition will become more intense with less equilibrium differences, and the quality-oriented operations strategy producer will ultimately gain. Thirdly, as the production cost gain of outsourcing increases, mainly through production cost increases in an outsourcer’s industry, outsourcing will become more attractive and the quality differences will be larger. Price differences at the equilibrium may vary depending on the direct effect of the production cost increases on pricing and the indirect effect of cost reduction of outsourcing. Fourth, if outsourcing requires additional monitoring of the outsourced production processes, the equilibrium rate of outsourcing will drop, leading to lowered price and quality differences (hence competition becomes intense). Within our framework, outsourcing brings expanded market share for the lower quality producer and hence better possibilities of profit gain.

Alternatively, any decrease (improvements) in outsourcing monitoring costs (e.g. through additional monitoring technology to improve efficiency or more careful selection of high quality suppliers (Aron et al., 2008)), improving market perceptions of outsourcing quality, or increases in production costs will increase outsourcing incentives and ultimately benefit cost-oriented producers by providing an improved competitive (relative market share and profitability) stance.
Well designed outsourcing may allow a firm to be more focused on strategically more important sets of tasks and to become more competent in its core capabilities. But it is also claimed that excessive outsourcing, without considering long term strategic consequences, may lead to unduly heavy dependence on subcontractors, undermining its R & D capabilities and causing potential technology predation by those suppliers (Wu et al (2005)). Advances in our model may require that we incorporate these other factors as potential strategic costs associated with outsourcing. Our model specifically addresses the issue that with quality competition in mind, this conclusion may be misleading, the lower quality producer by introducing and expanding the possibility of outsourcing may already committed to go for a lower quality acceptance of their product. This result does not mean that the industry as a whole will suffer from quality degradation. Rather, in fact, the high quality producer will actually put more labor for an upgrade, to go for greater differentiation as the quality producer in this market in the threat of outsourcing competitors. Hence, our results imply that outsourcing may actually provide customers with more quality product choice than before.

For a lower quality –cost-driven firm, this claim of higher quality through outsourcing decisions might be very valuable to be accounted for, but for an analyst more concerned about the whole picture of the consequences of outsourcing, the claim will be surprising, though it may describe the reality well enough. Thus, instead of just predicting whether organizations should outsource we find it is important to consider the problem within a framework of general quality competition rather than just explaining and predicting the outsourcing market for the sake of outsourcers themselves. The implications for operations strategy are also pretty clear here. Organizations seeking to outsource in a quality sensitive market should be wary of customer perceptions and that shifting to cost-based operations strategies may entail risks. Alternatively, for organizations in cost sensitive markets, having too much investment in quality initiatives may be detrimental to market share and profitability. An appropriate operations strategy balance is necessary depending on market characteristics, subcontractor capabilities, and organizational competencies.

Our approach to outsourcing with a three stage game theoretic model to provides significant insightful results, but there are some limitations, which provide ample
opportunity for future research. In our model, we implicitly assumed that the quality-oriented producer does not outsource, but it excludes many interesting cases from strategic interactions among the firms, including some dynamics. This may be a future task of our research.

References


Globe and Mail, 2007, Don’t bash China – U.S. toy makers are at fault, 3 Sept. Monday


Figure 1. Customer’s quality choice/perception ranges and utility values

\[ u_A = \theta_A v^1 + \theta - p_A \]

\[ u_B = \theta_B v^1 + \theta - \alpha r_B - p_B \]
Table 1. Numerical Analysis with respect to customer quality sensitivity to outsourcing ($\alpha$) (with $c = 2$, $\gamma = 1$)

<table>
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<tr>
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* In this table, $\theta_B^* = 0$ because of a corner solution to the optimal problem. This result is not surprising since we generally obtain the maximum quality differences in equilibrium in the vertical product differentiation literature due to introduction of a minimum level of quality, $\theta$, or $\theta_j$, is bounded from below. Table 2 and 3 show the similar results.
Table 1b: Numerical Analysis with respect to customer quality sensitivity to outsourcing ($\alpha$) (with $c = 2$, $\gamma = 1$).

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Table 2a. Numerical Analysis with respect to product input manufacture cost \((c)\) (\(\alpha = 1, \gamma = 1\))

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Table 2a. Numerical Analysis with respect to product input manufacture cost \((c)\) (\(\alpha = 1, \gamma = 1\))

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Table 3a. Numerical Analysis with respect to the level of monitoring costs ($\gamma$) ($\alpha = 1, c = 3$)

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